

Comment on “A distributed hydrology-vegetation model for complex terrain” by Mark S. Wigmosta, Lance W. Vail, and Dennis P. Lettenmaier

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Wigmosta et al. [1994] recently developed a spatially distributed hydrology-vegetation model to simulate interactions between various hydrologic processes and vegetation. Because few such integrated landscape-level models exist, this is an important and useful contribution to the hydrologic literature. Snow accumulation and melt are estimated in their model using a single-layer, energy and mass balance approach. Though we agree that processes of snow accumulation and melt can be simplified for computational efficiency, we noticed four instances in which better mathematical representations are possible and provide more realistic results. In this comment we use their symbol system wherever possible.

First, (26a) should be written as

$$\begin{aligned} P_s &= P & T &\leq T_{\min} \\ P_s &= \frac{T_{\max} - T}{T_{\max} - T_{\min}} P & T_{\min} &< T < T_{\max} \\ P_s &= 0 & T &\geq T_{\min} \end{aligned} \quad (1)$$

where a P has been inserted into the formulation.

Second, the reasoning behind (27) is unclear. The left-hand side of (27) is an energy change rate and the right-hand side is the total energy input to snowpack over the time interval (this can largely be deduced from equations after (29)). The same disparity in units applies to (28) as well. Another weakness related to (27) is the oversimplified assumption of water equivalent as constant; this leads to possible computational divergence of snow temperature during snow accumulation.

To correct this problem, we developed the following series of equations based on our concept of these processes and compared the results from our equations to those of *Wigmosta et al.* [1994]. We think a more precise formulation of the relationship presented in their equation (27) is

$$\frac{d(c_s W T_s)}{dt} = r_{ns} + q_s + q_e + q_p + q_m + q_g \quad (2)$$

where r_{ns} , q_s , q_e , q_p , q_m , and q_g are energy exchange rates; subscripts are as defined by *Wigmosta et al.* [1994], with the exception that we consistently use subscript e to denote latent heat of evaporation or condensation, whereas *Wigmosta et al.* use subscripts e and l interchangeably.

On the simplifying assumption that specific heat, c_s , is a constant, we can develop the following equation:

$$c_s \left[W \frac{d(T_s)}{dt} + T_s \frac{d(W)}{dt} \right] = r_{ns} + q_s + q_e + q_p + q_m + q_g \quad (3)$$

This is accomplished using the finite difference scheme:

$$\begin{aligned} W T_s^{t+\Delta t} + T_s^{t+\Delta t} \Delta W &= \frac{\Delta t}{c_s} (r_{ns} + q_s + q_e + q_p + q_m + q_g) \\ &+ W T_s^t \end{aligned} \quad (4)$$

Note that this approach includes a mass change term (ΔW) and assumes that the specific heat of ice (c_s) does not change with temperature (T_s).

If we ignore the mass change caused by latent heat transfer in the above equation (this is reasonable because the amount is relatively small compared to precipitation) and consider the mass change as produced solely by any precipitation, then $\Delta W = P$ and

$$T_s^{t+\Delta t} = \frac{\Delta t(r_{ns} + q_s + q_e + q_p + q_m + q_g) + c_s W T_s^t}{c_s(W + P)} \quad (5)$$

We can write this equation in terms of energy exchanges during the period t to $t + \Delta t$:

$$T_s^{t+\Delta t} = \frac{R_{ns} + Q_s + Q_e + Q_p + Q_m + Q_g + c_s W T_s^t}{c_s(W + P)} \quad (6)$$

where the Q terms are as defined by *Wigmosta et al.* [1994]. This equation is a modified version of (28). At isothermal conditions the heat for snowmelt can be obtained as

$$Q_m = -[R_{ns} + Q_s + Q_e + Q_p + Q_g + c_s W T_s^t] \quad (7)$$

which is essentially the same as (29) of *Wigmosta et al.* [1994]. The negative sign is implied by *Wigmosta et al.* [1994], but we express it explicitly.

The dW/dt term needs to be included in the energy budget equations (our equation (3)) since changes in snow water equivalent, W , with time cannot simply be ignored. Their approach will underestimate the snow temperature during seasons of snow accumulation. The following calculations demonstrate the difference between our approach and that of *Wigmosta et al.* [1994]. Assume there are 2 cm of snow water equivalent with temperature -2°C on the ground, and all of the energy exchanges are zero (except the advection term). Additional precipitation consisting of 2 cm of snow at air temperature -2°C occurs during a given period. Obviously in this case, the end temperature of snow should remain -2°C .

Advection heat input is

$$Q_p = 0.5 \text{ cal g}^{-1}\text{C}^{-1} (1 \text{ g cm}^{-3})(2 \text{ cm}^3)(-2^\circ\text{C}) = -2 \text{ cal}$$

The end temperature from *Wigmosta et al.* [1994, equation (28)] is

$$\begin{aligned} T_s^{t+\Delta t} &= T_s^t + \frac{R_{ns} + Q_s + Q_e + Q_p + Q_m + Q_g}{c_s W} \\ &= -2 + \frac{0 + 0 + 0 + (-2) + 0 + 0}{0.5(2)} \\ &= -4.0^\circ\text{C} \end{aligned}$$

The end temperature based on our equation is

$$\begin{aligned} T_s^{t+\Delta t} &= \frac{R_{ns} + Q_s + Q_e + Q_p + Q_m + Q_g + c_s W T_s^t}{c_s(W + P)} \\ &= \frac{0 + 0 + 0 + (-2) + 0 + 0 + 0.5(2)(-2)}{0.5(2 + 2)} \\ &= -2.0^\circ\text{C} \end{aligned}$$

Third, it is unclear whether the time interval has been overlooked in (30) and (31). Concerning the latent heat and sensible heat equations, equations (29), (35), and (36) imply that (30) and (31) are total sensible heat and latent heat, respectively, in which case a time term should be present. It is hard to imagine that the total sensible and latent heat for a period are independent of the length of the time period. We would think that (30) and (31) give us rates of heat flux rather than total heat exchange.

Last, minor problems also exist in their mass balance implementation. Equations (35) and (36) of *Wigmosta et al.* [1994] should be

$$\Delta W_{\text{liq}} = P_{\text{rain}} + \frac{Q_e}{\rho_w \lambda_v} - \frac{Q_m}{\rho_w \lambda_m} \quad (8)$$

$$\Delta W_{\text{ice}} = P_{\text{snow}} + \frac{Q_e}{\rho_w \lambda_s} + \frac{Q_m}{\rho_w \lambda_m} \quad (9)$$

where λ_v is the latent heat of vaporization, λ_m is the latent heat of melting, and λ_s is the latent heat of sublimation. We have put the correct latent heat terms in the above equations. Q_m should always refer to the latent heat of melting or freezing, and Q_e will relate to latent heat of vaporization in the snow-melting season and to latent heat of sublimation in the snow accumulation season.

References

Wigmosta, M. S., L. W. Vail, and D. P. Lettenmaier, A distributed hydrology-vegetation model for complex terrain, *Water Resour. Res.*, 30(6), 1665-1679, 1994.

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