Estimating Solar Radiation
Under Variable Cloud Conditions

DONALD R. SATTERLUND
JOSEPH E. MEANS

ABSTRACT. Measurement of solar radiation is not often feasible except at instrumented research facilities. Although estimates of direct beam solar radiation in the absence of an atmosphere or with cloudless skies may be obtained from theoretical equations or tables, they are unrealistic to the extent that they fail to consider scattered solar radiation and are unapplicable whenever clouds are present. This paper presents a simple model for estimating daily total solar radiation by modifying existing theoretical equations to take into account scattered radiation from clear skies and reflected and transmitted radiation from clouds. Observational data required are: station latitude, slope direction and inclination, and estimated mean hourly cloud cover, by month. Tests of estimated daily total solar radiation by months against comparable measured radiation for several areas in the Pacific Northwest indicate that the model yields estimates that reproduce measured daily totals with a high degree of accuracy. Results were similar when the model was used to estimate solar radiation in the northeastern United States. FOREST SCI. 24:363-373.

ADDITIONAL KEY WORDS. Cloud cover, model testing.

Scientists and resource managers are increasingly aware that solar radiation plays a central role in energy exchange processes that determine ecosystem distribution, composition, and productivity. Recurrent natural patterns reflect this climatic control both in seasonal and spatial development. Managers of forest, range, wildlife, and water resources must adjust their operations to these varying patterns, for similar practices yield different results as the solar radiation environment varies. Successful management therefore requires not only a firm understanding of the principles of energy exchange processes, but a knowledge of the solar energy environment of specific sites as well.

Unfortunately, measurement of solar radiation is seldom feasible except on instrumented research sites. Despite increasingly successful efforts to develop inexpensive, accurate instruments, many researchers and nearly all practicing managers must still depend upon estimates of incident solar radiation for their planning and operations.

Solar radiation is highly variable from place to place because of variation in exposure of the earth's surface to the sun and because of variable atmospheric characteristics. The characteristics of the surface and solar geometry determine the upper limit of solar radiation on any given site and atmospheric characteristics intercede to further limit the amount received.

The authors are Professor and undergraduate assistant, respectively, Department of Forestry and Range Management, Washington State University, Pullman 99164. J. E. Means is presently at the Forestry Sciences Laboratory, Pacific Northwest Forest and Range Experiment Station, Corvallis, Oregon 97331. Scientific paper 4627, Washington State University College of Agriculture Research Center, Pullman, project #0118. This investigation was supported in part by Cooperative State Research Service funds under the McIntire-Stennis program. Manuscript received 30 June 1976 and in final revised form 15 March 1978.
Estimates of potential direct beam solar radiation received on any site in the absence of an atmosphere, and of direct beam solar radiation with a clear atmosphere of varying transmissibility are readily computed (Fons and others 1960, Frank and Lee 1966, Garnier and Ohmura 1968, Furnival and others 1969, Buffo and others 1972). These authors also discuss the theory of calculating direct beam solar radiation. However, direct beam solar radiation is scattered in the atmosphere, producing diffuse clear sky radiation, and is transmitted through and reflected by clouds. Williams and others (1972) and Garnier and Ohmura (1968) have developed models which compute clear sky diffuse radiation. However, clouds intercept and reflect both direct beam and diffuse radiation, and are common nearly everywhere. It is therefore desirable to take the effect of clouds into account in the estimation of solar radiation climates.

Swift (1976) recently published a model that takes the total atmospheric effect into account, but it requires measurements of solar radiation on a nearby horizontal surface. The model described here requires only latitude, date, slope direction and inclination, and estimates of atmospheric transmissibility and cloud cover to provide an estimation of the daily total solar radiation received at a surface. Thompson (1976) also published an empirical method of estimating solar radiation in relation to mean daily sky cover. Results of his method are compared with those obtained in this study in the Pacific Northwest to illustrate the improvement in estimates possible with the new model.

**Direct Beam Solar Radiation.**—The instantaneous radiation at a station on the earth’s surface with an arbitrary slope is given by Fons and others (1960):

\[ I_{D8} = I_0 P \cos \theta \sin \theta \]  

(1)

where \( I_{D8} \) = direct beam solar radiation incident on a unit surface with an arbitrary slope (ly/min),
\( I_0 \) = direct beam solar radiation on a surface normal to the sun in the absence of an atmosphere. (2.0 ly/min) (it is called SCON in the program and can easily be changed),
\( P \) = transmission coefficient for the atmosphere,
\( \phi \) = angular altitude of the sun from the horizontal,
\( \theta \) = angle between the surface and the radiant beam.

Angle \( \phi \) varies as the relation between the earth and the sun according to position of sun north or south of the celestial equator, latitude of the surface, and time of day (Fons and others 1960):

\[ \sin \phi = \cos \delta \cos h \cos \phi + \sin \phi \sin \delta \]  

(2)

where \( \delta \) = declination of the sun from the celestial equator,
\( h \) = hour angle of the sun from 1200 solar time,
\( \phi \) = latitude of the surface receiving radiation.

Angle \( \theta \) was determined by the equivalent point technique outlined by Frank and Lee (1966).

\[ \sin \theta = \sin \phi' \sin \delta + \cos \phi' \cos \delta \cos (L + h) \]  

(3)

The equivalent latitude, \( \phi' \), is given by:

\[ \sin \phi' = \sin k \cos g \cos \phi + \cos k \sin \phi \]  

(4)

where \( k \) = slope inclination,
\( g \) = slope azimuth, measured clockwise from north.
\[ L \text{ is the equivalent longitude.} \]

\[ \tan L = \sin g \sin k \left( \cos k \cos \phi - \cos g \sin k \sin \phi \right). \] \hspace{1cm} (5)

With the choice of an appropriate atmospheric transmission coefficient \((P)\) equation (1) can be solved to estimate the instantaneous direct beam solar radiation values from a cloudless sky for any plane surface. Equation (1) provided the basis for our extended model, which was designed to modify the direct beam solar radiation model by taking clouds and scattered clear sky radiation into account.

**Solar Radiation and Clouds.**—Clouds intercept solar radiation, but do not completely prevent it from reaching the ground. A variable portion is reflected and transmitted, depending on cloud type, thickness, altitude, and other factors.

Three main problems had to be met if clouds were to be taken into account: (i) sky cover observations must be available, (ii) the quantity of reflected and transmitted radiation must be established, and (iii) the distribution of reflected and transmitted solar radiation upon slopes must be taken into account.

Airport observers and a few other agencies report observed cloud cover taken at intervals of several hours usually resolving the cloud cover in tenths on a scale of 0–1. Cloud altitude is sometimes given. Compilation of such data for the Columbia Basin States, including the mean tenths of cloud cover by 3-hour periods, by months, for airports and several mountain passes for the years 1949–1958 is available from the Pacific Northwest River Basins Commission (1968). Graphical interpolation was used on these data to obtain hourly estimates of cloud cover.

Gates (1962) and Reifsnvnder and Lull (1965) reviewed methods of estimating the effects of clouds on solar radiation and Thompson (1976) developed a model taking clouds into account. Many of the equations were regressions based on mean daily cloudiness or percent of possible sunshine. They provided reasonable estimates of solar radiation received on horizontal surfaces, but the use of mean daily data would mask the effects of diurnal variation in cloudiness on radiation receipt on different slopes as pointed out by Geiger (1959). Local data revealed that mean hourly cloud cover, by months, in the Pacific Northwest varied by as much as four-tenths at some coastal stations between morning and afternoon, and frequently exceeded two-tenths in the interior (Pacific River Basins Commission 1968). Therefore it appeared necessary to increment the program in hourly steps.

More sophisticated empirical models and measured data indicate that equivalent air mass depth; cloud type, thickness, and altitude, all influenced reflected and solar radiation from clouds (Gates 1962, Sivkov 1968). However, detailed data of this type are not widely available and unlikely to become so. Nevertheless, the prevalence of low, stratiform clouds in winter and higher, cumuliform clouds in summer suggested that these factors should be considered.

Therefore, first efforts to model the effect of clouds were based on mean hourly cover, and attempted to develop parameters that would take seasonal effects on cloud type, cloud thickness, and cloud elevation into account. The general form of the model was:

\[ I_c = I_{DO} C e^{-d} \] \hspace{1cm} (6)

where
- \(I_c\) = transmitted and reflected solar radiation from clouds (ly/min),
- \(I_{DO}\) = total solar radiation on a horizontal surface in the absence of clouds (ly/min),
- \(I_{DO} = I_{DS}\) when \(k = 0\), so,
- \(I_{DO} = I_oP^{(eSC.A)} \sin A\),
- \(C\) = cloud cover, expressed as a decimal,
- \(d\) = derived parameter to express seasonal cloud effects.
Kondratyev (1969) gives a correction factor for scattered clear sky radiation on a slope as:

\[ F = \cos^2 \left( \frac{k}{2} \right) \]  

(7)

Since transmitted and reflected radiation from clouds is here assumed to be non-directional, as is scattered clear sky radiation (ignoring anisotropy), this correction is used for transmitted and reflected radiation from clouds as well as for scattered clear sky radiation. Thus transmitted and reflected radiation from clouds on a slope \( = I_c F \).

**Scattered Solar Radiation From Cloudless Skies.**—Scattering of solar radiation in the atmosphere is highly complex. Part of the direct beam radiation that is depleted by scattering does reach the ground, but is without its directional component. Depletion of direct beam solar radiation is primarily a function of transmissibility of the atmosphere (in turn a function of dust, haze, and other characteristics), and the air mass through which the rays pass. In the direct beam solar radiation model of Fons and others (1960), these factors are expressed as a function of \( P_{\text{cse}} A \), where \( p \) = an atmospheric transmissibility coefficient, and \( \text{csc} A \) = an expression of the approximate effective depth of the atmosphere.

It seemed reasonable that the same atmospheric characteristics that cause depletion of direct beam solar radiation by absorption and scattering should also determine the amount of scattered clear sky radiation reaching the surface. Clear sky scattered radiation reaching the surface represents only a part of scattered solar radiation, because part is absorbed in the atmosphere and part is reflected back to space. Therefore, two working assumptions were made: (i) that total solar radiation (direct beam and scattered) reaching a horizontal surface from a clear sky should not on the average, exceed potential solar beam radiation, and (ii) the quantity of scattered radiation should be some function of atmospheric transmissibility. Data from Brooks are presented in Reifsnyder and Lull (1965) in which scattered and total solar radiation received on a horizontal surface for a standard cloudless atmosphere and an industrial cloudless atmosphere supported these assumptions and set approximate limits on the receipt of scattered solar radiation as a function of solar altitude.

The general model chosen was:

\[ I_s = I_0 e^{-aP_{\text{cse}} A} b P_{\text{cse}} A \]  

(8)

where \( I_s \) = scattered clear sky solar radiation on a horizontal surface (ly/min), \( P_{\text{cse}} A \) = transmissibility of equivalent air mass distances and turbidity, \( a, b \) = derived empirical coefficients.

Tests against the data shown in Reifsnyder and Lull (1965) gave a best fit with \( a = 2 \) and \( b = 0.35 \) for a standard cloudless atmosphere and zero slope. Therefore, these values were chosen for the model. Scattered clear sky radiation on a slope \( = I_s F \).

**The Total Solar Radiation Model**

The three elements, (i) direct beam solar radiation, (ii) scattered solar radiation from clear skies, and (iii) reflected and transmitted solar radiation from clouds were combined to obtain instantaneous estimates of solar radiation for each daylight hour of the mid-date of each month.

\[ I_T = (I_{D8} + I_s F) (1 - C) + (I_{DO} + I_s) C e^{-a F} \]  

(9)

where \( I_T \) = solar radiation received on any unit surface (ly/min),
\[ I_{DS} = \text{direct beam solar radiation received (varies by slope inclination and aspect)} \ (\text{ly/min}), \ \text{equation (1)}, \]
\[ I_{DO} = \text{direct beam radiation on horizontal surface (ly min),} \]
\[ I_{S} = \text{scattered clear sky solar radiation (ly min), equation (8).} \]

The estimates of instantaneous solar radiation at each hour (ly/min) were integrated using a trapezoidal approximation, yielding output of mean hourly and, by summation, mean daily solar radiation for the period between sunrise and sunset for the mid-date of each month for each station. The method of summing hourly values to obtain mean daily solar radiation automatically compensates for the variable effect of clouds at different hours of the day. Thus, clouds occurring near the time of sunrise or sunset (when incoming solar radiation is low regardless of sky cover) have less effect on daily solar radiation than do clouds near noon (when the rate of incoming solar radiation is much greater). Similarly, clouds in the morning have a greater effect on daily solar radiation received on steep easterly slopes than would the same amount of cloud cover in the afternoon. The value of parameter \( P \) (atmospheric transmissibility) and \( d \) (empirical constant of eq. 6) were selected to give the best fit to sixty station months of measured mean monthly solar radiation (U.S. Weather Bureau 1964) at five airports (Astoria and Medford, Oregon; Boise, Idaho; Seattle-Tacoma and Spokane, Washington) representative of the range of the solar radiation climate of the Pacific Northwest.

**RESULTS AND DISCUSSION**

The first few runs of the model failed to yield any reasonable estimate of measured mean daily solar radiation. Graphs of the output indicated estimates were too low under conditions of moderate sky cover when they fit extremes of clarity or cloudiness, and too high at the extremes when they fit intermediate conditions. It appeared that something was wrong with the model or the cloud cover data. Since the model appeared to fit both extremes of cloudiness or clear skies, we first decided to examine the cloud cover data.

Two types of sky cover data are commonly available: observations of mean sky cover without regard to cloud type, thickness, or altitude, and sunshine recorder measurements. When mean daily sky cover between the hours of sunrise and sunset was plotted against sunshine recorder measurements (Pacific Northwest River Basins Commission 1968) a curvilinear relationship was evident (Figs. 1 and 2). Winter and spring observations were more strongly curvilinear than summer and fall observations. The probable reasons include the greater frequency of cumulus or thin and/or high altitude clouds that transmit or redirect sufficient direct beam solar radiation to register on the sunshine recorder in summer and fall than in winter and spring. Yet, such clouds are clearly evident to an observer on the ground and are included in the estimate of sky cover.

Therefore, an adjustment equation based on the Thompson (1976) model was developed so that sky cover observations conformed with sunshine recorder estimates. The equation used was:

\[
Y = 1 - [B + (1-B)(1-X)^K] \quad (10)
\]

where \( Y = \text{adjusted cloud cover,} \)
\( B = \text{an empirical constant (-0.90 in winter and spring, -0.95 in summer and fall),} \)
\( X = \text{observed cloud cover, and} \)
\( K = \text{an empirical constant (0.24 in winter and spring, 0.29 in summer and fall).} \)

The model was then tested again using adjusted cloud cover observations.
An excellent fit was obtained (Fig. 3) using values for $P$ (eq. 1) of 0.76 during May through September and 0.80 during the rest of the year, and for $d$ of 1 throughout the year. The computed values of mean daily radiation had a standard error of measured from estimated values of 15.58 langleys (65.21 joules/cm$^2$) and 7.26 percent over a wide range of measured values (64-124 ly/day, 268-519 joules/cm$^2$/day) for December and 539-698 ly/day (2256-2921 joules/cm$^2$/day) for July). A forced fit of the model proves little about its predictive value in other situations, but the size of the deviations suggested that the basic model was valid. It remained to be seen how good estimated radiation would be using completely independent data.

The best available data in the region were utilized to develop the model and select parameters. However, there existed two sets of short-term solar radiation data that were measured within 30 miles of airport cloud observations: Friday Harbor and Bellingham Airport, Washington, and Corvallis and Salem Airport, Oregon, 20 and 25 miles apart, respectively. Accordingly, these two sets of data, completely independent of those used to derive the model, were used to test it.

The fact that individual observations of each pair were conducted at sites somewhat separated in space was not considered a serious deficiency. Both pairs were within a relatively homogeneous physiographic and climatic locality where solar radiation would not be expected to vary greatly. Furthermore, if the model is to be useful, it must be applicable at some distance from the source of the basic cloud cover observations that are available only at airports and a few mountain passes.

The results of the independent test considerably exceeded our expectations (Fig. 4), being nearly identical in fit as the original from which the parameters were derived, despite the separation of the sites of paired observations. The standard error was 19.60 langleys (82.04 joules/cm$^2$) and 6.50 percent.

These exceptional results require explanation, for the model is simple, and cannot physically account for all the varying and complex effects of the atmosphere on...
solar radiation. It is possible that chance contributes to the exceptional results, but that chance alone is involved must be rejected on both statistical \((r = 0.9954)\) and logical grounds. The core of the model is the deterministic geometrical relation between the earth and sun. To determine how much of these results represented an improvement over the use of an empirical solar radiation model that includes the effect of clouds a simple regression model of measured versus estimated solar radiation (Thompson 1976) was compared with our model.

The model presented here reduced the difference between measured and estimated solar radiation by a factor of 2.28 times (Table 1) from that of the Thompson (1976) model when all of the data are combined. Further, both the slope of the equation, and its y intercept were changed in a manner that suggests a more direct relation between estimated and measured solar radiation on horizontal surfaces.

That the good fit of the model is perhaps somewhat fortuitous is evident from a comparison of two sets of measured mean daily solar radiation data, by months at Spokane, Washington (each value was an average of a number of yearly values). The first set consisted of all usable data (7 to 10 years for each month) through 1962 (U.S. Weather Bureau 1964); the second consisted of 10 years of data taken from 1955 through 1964, except for October 1959 and November and December 1963 (Phillips 1965). Despite the fact that the two sets of data included 5 to 8 years in common, depending on month, they differed by as much as 28 ly/day (119 joules/cm²/day) and up to 8.4 percent for individual months.

The data available are limited and this factor limited the number of refinements it was possible and profitable to build into the model. There is variation in the cloud cover data as shown in Figures 1 and 2 and in the solar radiation data as evidenced above.
In view of the limitations of the model and the limited data bases from which it was derived and against which it was tested, one might be tempted to question its utility to predict mean solar radiation conditions elsewhere. Consequently, cloud cover observations and mean daily solar radiation measurements, by months, were obtained for six stations in northeastern United States: New York City and Ithaca, New York; Burlington, Vermont; Portland, Maine; and Boston and Blue Hill, Massachusetts (U.S. Weather Bureau 1962–63, 1964, 1960–71). The model, using coefficients derived in the Pacific Northwest was tested using these data. Estimated solar radiation values were consistently high for stations in the Northeast. However, a slight reduction in the transmissibility coefficient from 0.80 to 0.75 (0.73 for the urban stations Boston and New York) in summer, and use of winter–spring cloud adjustment for the period October through April resulted in an excellent fit of estimated and measured solar radiation (Fig. 5). With these minor adjustments, the model estimated solar radiation as well in the Northeast as in the Pacific Northwest (Table 1) with a standard error of 12.04 ly/day (50.39 joules/cm²/day) or 5.86 percent.

From these tests we conclude that the model developed here is capable of estimating mean daily solar radiation on a horizontal surface with a high degree of accuracy over a wide range of atmospheric conditions. The estimates can be extrapolated from the site of cloud cover observations so long as they are kept within a relatively homogeneous physiographic and climatic area.

Coefficients used in the model probably should be developed separately for each region. For example, the downward adjustment in the transmissibility coefficient in the Northeast as compared to the Northwest may reflect a more polluted atmo-
TABLE 1. Statistical comparison of measured versus estimated solar radiation, based on regressions with two estimating models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Regression parameters</th>
<th>Standard error of regression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( y = a + bx )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( a )</td>
<td>( b )</td>
</tr>
<tr>
<td>Present model, data set 1</td>
<td>0</td>
<td>1.005</td>
</tr>
<tr>
<td>Present model, data set 2</td>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>Present model, data set 3</td>
<td>0</td>
<td>1.002</td>
</tr>
<tr>
<td>Thompson (1976) model, data set 1</td>
<td>-42.65</td>
<td>1.160</td>
</tr>
<tr>
<td>Thompson (1976) model, data set 2</td>
<td>-47.08</td>
<td>1.214</td>
</tr>
<tr>
<td>Thompson (1976) model, data set 3</td>
<td>-43.17</td>
<td>1.172</td>
</tr>
<tr>
<td>Present model, Northeastern U.S.</td>
<td>3.11</td>
<td>0.989</td>
</tr>
</tbody>
</table>

Data set 1 is that used to derive parameters of model \( n = 60 \), set 2 is that used in independent test \( n = 23 \), set 3 is sets 1 and 2 combined \( n = 83 \), as sets 1 and 2 were not significantly different. Northeastern U.S. \( n = 72 \).

sphere in the former region, and especially in large urban areas such as New York and Boston. Similarly, seasonal cloud characteristics may vary with large scale differences in air mass characteristics in different parts of the continent.

A question remains, however, whether the model is valid for all slopes and aspects when applied to mountainous topography. No data are available to answer this question unambiguously. A positive answer rests on two factors: (i) the proper partitioning of total solar radiation into direct beam and scattered components, and (ii) the magnitude of the effect of shading by nearby hills which the model does not take into account.

Possible error in partitioning solar radiation into direct beam and scattered components holds a significant potential for invalidating the model. The tests of the model against measured radiation on horizontal surfaces are suggestive only, but they do indicate that the model properly partitions the solar radiation.

Two factors support this inference: first, the form of the regression equation of measured versus estimated radiation based on this model is \( y = bx \). This means that even in the absence of direct beam solar radiation, as on a steep north-facing slope in winter, the model predicts the receipt of a small amount of scattered radiation. In all tests the value of the \( y \) intercept was not significantly different \( (p = 0.05) \) from 0.

Second, if partitioning were substantially in error, there should be a noticeable difference in the accuracy of estimated radiation from place to place and season to season, for mean hourly cloud cover variations, by month (adjusted), ranged from 0.88 to 0.11. No consistent locational or seasonal variation in the relation between estimated and measured solar radiation was evident.

Topographic shading may be important in mountainous areas (Williams and others 1972). However, shadow boundaries can easily be located for any time and shadow source (Satterlund 1977), and since the model output includes hourly values, adjustment for topographic shading is straightforward.
In conclusion, it appears possible to obtain reasonably accurate estimates of the daily total solar radiation from data on location, slope aspect and inclination, and mean diurnal cloud cover. Cloud cover data are collected at many weather stations so the model should be applicable over a large area. The model could possibly be extended to other regions than northeastern United States although it is probable that different coefficients would need to be developed to take differences in cloud types, altitude, and atmospheric turbidity into consideration. The major limitation is the simplicity of the model and restricted data on cloud cover. It would be unjustified at this time to use the model to estimate solar radiation for a particular cloud cover condition or a particular day because the model gains much of its present accuracy from the use of mean monthly diurnal data. The model output is an estimate of mean of solar radiation received at a surface, not its precise value at any given time.

LITERATURE CITED


