Jinfan Duan, Forest Engineering Department, Oregon State University, Corvallis, Oregon 97331

Alok K. Sikka, Forestry Sciences Laboratory, Oregon State University, Corvallis, Oregon 97331

and

Gordon E. Grant, Pacific Northwest Research Station, U.S. Forest Service, Corvallis, Oregon 97331.

A Comparison of Stochastic Models for Generating Daily Precipitation at the H. J. Andrews Experimental Forest

Abstract

Estimates of precipitation are often required to model hydrologic and ecologic processes. A variety of methods can be found in the literature for modeling the occurrence and distribution of daily precipitation amounts. Here we test their applicability against empirical data for the H. J. Andrews Experimental Forest in Oregon. A stochastic model for generating daily precipitation was developed using a two state Markov chain model for occurrence of precipitation. Five different theoretical distributions for the precipitation amount were compared, using data from the Andrews Experimental Forest in the Western Cascades of Oregon. Of the distributions examined (exponential, calibrated Weibull, calibrated beta-P, and two 2-parameter gamma distributions), the calibrated Weibull distribution and two parameter gamma distribution appear to be better for simulating daily precipitation amounts. Such models can be used to generate precipitation input to process-based deterministic hydrologic models useful for simulating hydrologic and ecologic responses to cumulative watershed effects.

Introduction

Stochastic models for generating daily precipitation are frequently used in a variety of hydrologic, ecologic, geomorphic, and water resources studies. These models rely on assumptions about the probability distribution of rainfall for a particular landscape. Such models are useful for extending meteorologic and hydrologic records beyond the timeframe of empirically measured values, filling in missing data at long-term measurement sites, and synthesizing records for unmeasured watersheds. The output from stochastic precipitation models can be used, in turn, as input to hydrologic and ecologic models that predict soil moisture, runoff, or the availability of water for plants.

Here we describe developing a model for generating precipitation data as input to process-based deterministic hydrologic models at the H. J. Andrews Experimental Forest (HJAEF). The Forest, which is located in the western Cascades of Oregon, and is a Long Term Ecological Research (LTER) site funded by the National Science Foundation (NSF). A major topic of research at the HJAEF is the relationship among hydrologic, ecologic, and geomorphic processes. While precipitation and other climate data has been recorded for more than 50 years at the Andrews, models to simulate precipitation are needed to extend that record over century or longer timescales associated with forest sucession, disturbance, and other ecological processes.

A variety of models can be found in the literature for simulating daily precipitation (Todorovic and Woolhiser, 1974; Smith and Schreiber, 1974; Chin, 1977; Buishand, 1978; Roldan and Woolhiser, 1982; Richardson and Wright, 1984). Due to the complex nature of rainfall processes, any rainfall model involves simplified assumptions of the process. For most practical problems, the model that best describes the precipitation distribution and amounts is preferred (Richardson, 1982). Our purpose here is to test a variety of models with different underlying assumptions against the actual data in order to select the best model or models to use for the HJAEF. We expect that the models that work best for the HJAEF will also be most appropriate for other watersheds in the western Cascades with similar precipitation regimes.

Models for generating daily rainfall sequences typically use a separate process for the rainfall occurrence (wet-dry event) and another process for the rainfall amounts on wet days (Todorovic and Woolhiser, 1974; Buishand, 1978; Richardson, 1982). Several approaches have been used for describing the occurrence of rainfall and the distribution of rainfall amounts given the occurrence of rain (Buishand, 1978; Roldan and Woolhiser, 1982; Woolhiser and Roldan, 1982; Richardson, 1982). For the rainfall occurrence, the Markov chain and the alternating renewal process (ARP) have been generally used for modeling wet- dry day sequences. In particular, the Markov chain model is used extensively for daily precipitation (Gabriel and Neuman, 1962; Haan et al., 1976; Smith and Schreiber, 1974; Roldan and Woolhiser, 1982; Richardson and Wright, 1984). Buishand (1978), and Roldan and Woolhiser (1982) reported advantages of using the Markov chain model over the ARP model. They reported that the generation of synthetic sequences is simpler for a Markov chain and its parameters can be obtained more easily than for the ARP model.

The most common approach for describing the distribution of rainfall amounts on days with rain is to ignore the serial autocorrelation and consider that rainfall amounts are serially independent and to fit some theoretical distribution to the precipitation amounts (Todorovic and Woolhiser, 1974, 1975; Woolhiser et al., 1973; Smith and Schreiber, 1974; Richardson, 1982). Various probability density functions ranging from single parameter to multiple parameters have been proposed to describe the distribution of rainfall amounts (Todorovic and Woolhiser, 1974, 1975; Smith and Schreiber, 1974; Richardson, 1982; Woolhiser and Roldan, 1982; Pickering et al., 1988). Woolhiser and Roldan (1982) compared the chain-dependent (i.e. precipitation amounts are independent but the distribution depends on the state of the previous day) and independent exponential, gamma, and three parameters mixed exponential distributions and showed that the independent mixed exponential was the best for five U.S. stations. It should be noted that the single parameter models are more appealing for their simplicity and ease of parameterization. However, there is no general consensus on the performance or applicability of a specific model; choice of an appropriate model depends on the underlying distribution for the observed data. This suggests a need of examining different distributions and building a precipitation model for a site.

In this study, a first-order Markov chain with two states, wet or dry, was used for simulating the occurrence of precipitation. A continuous distribution function was used to model the amount of precipitation given that a day is wet. To accomplish this, five probability distributions for the precipitation amount (the exponential, calibrated Weibull, calibrated beta-P, and 2-parameter gamma distribution with two different estimators) were compared, using data from the HJ Andrews Experimental Forest in the Western Cascades of Oregon. The performance of these models in simulating daily and monthly precipitation at the Andrews Forest is presented.

The Models

Occurrence Process

Occurrence of precipitation is described by a two state Markov chain (day is wet or dry) of first order; that is, the probability of precipitation on a given day depends solely on whether or not precipitation occurred on the previous day. This approach has been used successfully and studied extensively to generate rainfall (Bailey, 1964; Haan, 1977; Richardson, 1981; Roldan and Woolhiser, 1982). The transition matrix is defined by:

$$P=\begin{bmatrix} p_{dd} & p_{dw} \\ p_{wd} & p_{ww} \end{bmatrix} \qquad (1)$$

where p_{dd}, p_{dw}, p_{wd}, p_{ww}, are the conditional probabilities of a dry day following a dry day, a wet day following a dry day, a dry day following a wet day and a wet day following a wet day, respectively. A day with total rainfall of 0.0254 centimeters (0.01 inch) or more was considered a wet day. Since $p_{dd} + p_{dw}=1$ and $p_{wd} + p_{ww}=1$, P is fully defined given p_{dw} and p_{ww}. The occurrence is determined by comparing a generated random number with the elements of the probability transition matrix. If the previous day is dry, then a generated random number less than p_{dw} represents rain on the current day; and if the previous day is wet, a generated random number less than p_{ww} means the current day is also wet. Otherwise, the current day is a dry day.

Precipitation Amounts

Rainfall amounts are considered serially independent. Five different models of the distribution of daily rainfall amounts are used and compared. Seasonal variations of precipitation are determined by assuming that the model parameters are constant within a month and different between months.

Single parameter models

The exponential distribution is probably the most widely used single parameter model of daily precipitation amounts for its simplicity, invertibility and relatively good fit (Todorovic and Woolhiser, 1974; Richardson, 1981; Pickering et al., 1988). The cumulative distribution function of the exponential is given by

$$F(x)=1-e^{-\frac{x}{\lambda}} \tag{2}$$

Where x is the daily precipitation, F(x) is the probability of events less than x, and $\lambda=E(X)$ is expectation of daily precipitation.

Single parameter probability distributions have also been derived by calibrating multi-parameter distributions (Pickering et al., 1988; Selker and Haith, 1990). Pickering et al. (1988) derived a single parameter model by calibrating a special case of the beta-P distribution model. In its most general form, beta-P is a three parameter model. The cumulative distribution function of the calibrated beta-P model of Pickering et al. (1988) is given by:

$$F(x)=1-(1+\frac{x}{9\lambda})^{-10} \tag{3}$$

A member of the Weibull family of distributions was given by Rodriguez (1977) as:

$$F(x)=1-e^{-[\frac{\Gamma(1+\frac{1}{c})x}{\lambda}]^{c}} \tag{4}$$

where c is a constant and $\Gamma(.)$ denotes complete gamma function.

The calibrated Weibull distribution was presented by Selker and Haith (1990) based on the above equation. With c=0.75, as the optimized value for the data from Eastern USA, they derived the probability distribution for wet day given by:

$$F(x)=1-e^{-(1.191\frac{x}{\lambda})^{0.75}} \tag{5}$$

In this study, we calibrated this model using Andrews Forest data to obtain optimized values of the parameters, which is discussed in the next section.

Multi-parameter models

Multi-parameter distributions such as the 2-parameter gamma, the 3-parameter gamma, the 3-parameter mixed exponential, and others have been used (Mielke and Johnson, 1974). Multi-parameter models are generally considered to describe the distribution of precipitation amounts better than the one-parameter exponential distribution because of the greater flexibility obtained with the large number of parameters. The choice of model depends on the parsimony of parameters and ease of estimating parameters. Richardson (1982) suggested that unless the mixed exponential distribution has a clear advantage over the 2-parameter gamma distribution, the gamma distribution should be the appropriate choice of models for most applications.

The 2-parameter gamma distribution is used here with the general form of the probability density function given by:

$$f(x)=\frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)} \tag{6}$$

where α and β are constants in gamma distribution. The gamma distribution is used with two different methods of parameter estimation. One method uses moment estimators (Devore, 1987), given by:

$$E(X)=\alpha\beta \tag{7a}$$

$$V(x)=\alpha\beta^{2} \tag{7b}$$

where E(.) denotes the expectation and V(.) the variance of the data. The other method uses maximum likelihood estimators, with:

$$\alpha\beta=E(X) \tag{8a}$$

$$Y=\ln(E(X))-E(\ln(X)) \tag{8b}$$

where Y is a intermediate value to be used in the following calculations.

There are a number of numerical schemes for determining the maximum likelihood estimators for the 2-parameter gamma (Thom, 1958; Greenwood and Durand, 1960; Mielke and Johnson, 1974; Choi and Wette, 1969). The numerical solution suggested by Choi and Wette (1969) was used in this study, which approximates the solution by Newton-Raphson iteration using the following equations:

$$\alpha_{i}=\alpha_{i-1}-\frac{\log(\alpha_{i-1})-\Psi(\alpha_{i-1})-Y}{\frac{1}{\alpha_{i-1}}-\Psi'(\alpha_{i-1})} \tag{9a}$$

$$\Psi(\alpha) = -\gamma - \frac{1}{\alpha} + \alpha \sum_{i=1}^{\infty} \left(\frac{1}{i(i+\alpha)} \right) \quad (9b)$$

$$\Psi'(\alpha) = \sum_{i=0}^{\infty} \frac{1}{(i+\alpha)^2} \quad (9c)$$

where γ=Euler's Constant=0.57722157 and Ψ is Digamma function.

The above infinite summations are approximated using an arbitrary 10^{-7} criterion. If the difference between calculated value at time i and time i-1 is less than the criterion, then the value at time i is used. Since this numerical solution is always convergent with any initial value such that $0<a_0<\infty$(Choi and Wette, 1969), an arbitrary number is given as a starting value in the computational estimation.

Experimental Design

The precipitation data of climatological station at Watershed #2 from H. J. Andrews Experimental Forest was used to calibrate and test the models since it has the longest observation history. The HJAEF is a mountain watershed of 6400 ha. Climate of the HJAEF is wet and fairly mild in the winter and warm and dry in summer, with more than two-thirds of the precipitation falling between November through March (Bierlmaier and McKee, 1989). Climatological information has been collected at the HJAEF since 1951. Daily precipitation data from 1952 to 1992 was used in this analysis. Although precipitation can occur as rain or snow, this paper does not differentiate among these forms and only considers the liquid water equivalent.

For the calibrated Weibull distribution, a set of new parameter values for the HJAEF were obtained using a least Chi-square criteria. Simulations were made with a range of parameter (c) values between 0.5 and 1.5, with a step value of 0.01, and the Chi square values were computed. The parameter value which gave the least Chi square was selected as an optimized value (Figure 1). The following equation for the calibrated Weibull distribution was then used in this study:

$$F(x) = 1 - e^{-(1.052\frac{x}{\lambda})^{0.9}} \quad (10)$$

In this study seasonal variations in the daily precipitation processes are assumed to be constant within a month and vary between months. Parameters are calculated based on monthly data; twelve sets of parameters for the five distribution

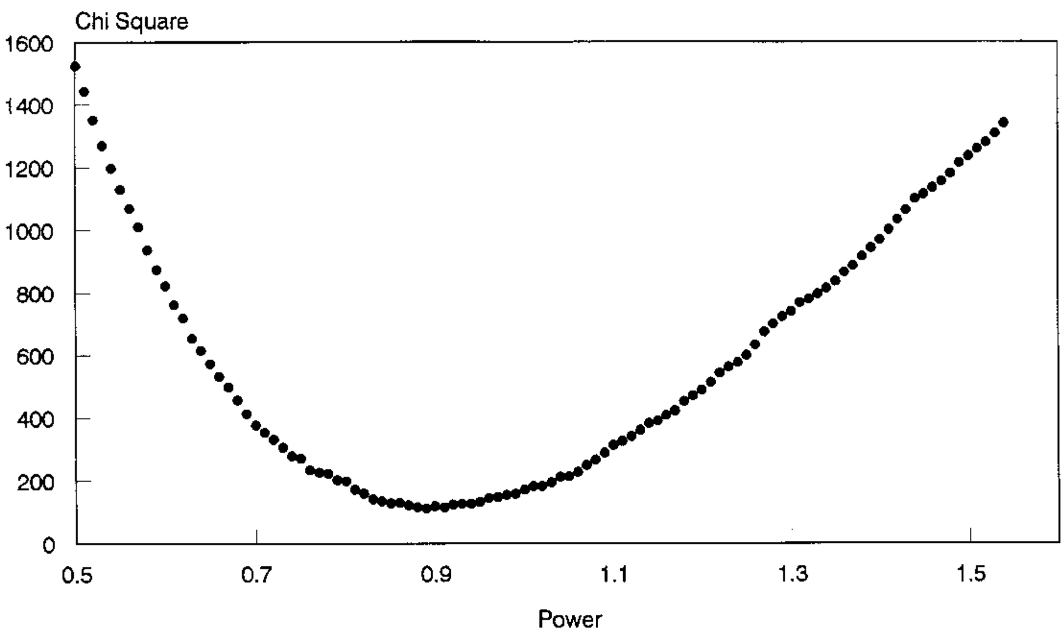

Figure 1. Calibration for constant *c* in general Weibull equation based on Chi Square. Note minimum Chi square is at 0.90.

models were therefore obtained. Although there are complex techniques, such as finite Fourier series, to model seasonal variations (Roldan and Woolhiser, 1982), the simple technique used in this paper is reported to give fairly good results of distributions for wet days (Richardson, 1981). A summary of model parameters based on the 41 years of data is given in Table 1. The above precipitation models were then run for 41 years with five distributions of precipitation amounts. Daily amounts were summed to obtain monthly and annual precipitation amounts.

A Chi Square test was used to test and quantify the goodness-of-fit of the above models. For daily and monthly analyses, data were categorized into 51 bin cells. It was made sure that each bin cell possessed at least 5 occurrences. A Chi-square tested whether the two data sets came from the same population by:

$$\chi^2 = \sum_{i=1}^{n} \frac{(OBSERVED_i - SIMULATED_i)^2}{OBSERVED_i + SIMULATED_i} \qquad (11)$$

where SIMULATED and OBSERVED are number of events in a combined bin i; n is the total number of bins constructed.

Since each term in a Chi-square sum is supposed to approximate the square of a normally distributed quantity with unit variance, the variance of the difference of two normal quantities is the sum of their individual variances, not the average. So the denominator of the above Chi-square was modified from the regular equation (Press et. al., 1992).

The probabilities of the observed data and the simulated data belonging to same distribution were calculated using the equation:

$$Prob = \frac{\Gamma\left(\frac{n}{2}, \frac{\chi^2}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} \qquad (12)$$

where Γ(...) denotes incomplete gamma function. A better model should give lower Chi-square values and high probability values. Table 2 shows the results of the Chi-square test used for daily and monthly data.

Results and Discussion

The analysis of 41 years of observed precipitation data from the HJAEF shows that about 55% of days were dry days (precipitation less than 0.01 inch). The distribution of relative frequencies of daily rainfall amounts, as shown in Figure 2, has a reversed J-shaped exponential distribution with the smaller amounts occurring more frequently than the larger amounts. Percentage of days with precipitation was highest from October though March (about 60%), with the wet day average

TABLE 1. Parameters of H. J. Andrews Precipitation Model.

Month	Daily Mean (cm)	Variance (cm)	Prob. of Dry to Wet	Prob. of Wet to Wet	Prob. of Wet Day	α for Moment Estimator	β for Moment Estimator	α for Maximum Likelihood Estimator	β for Maximum Likelihood Estimator
1	1.791	1.514	0.327	0.816	0.64	0.833	0.846	0.914	0.771
2	1.575	1.171	0.345	0.785	0.616	0.835	0.743	0.901	0.688
3	1.339	0.683	0.336	0.793	0.619	1.032	0.511	1.027	0.513
4	0.937	0.465	0.319	0.741	0.552	0.744	0.496	0.959	0.385
5	0.838	0.282	0.223	0.680	0.410	0.982	0.336	1.069	0.308
6	0.709	0.279	0.168	0.599	0.295	0.709	0.394	0.988	0.283
7	0.488	0.137	0.064	0.466	0.107	0.684	0.281	1.034	0.186
8	0.729	0.208	0.074	0.584	0.150	1.002	0.286	1.109	0.259
9	1.046	0.505	0.125	0.607	0.241	0.854	0.483	0.946	0.436
10	1.420	0.973	0.199	0.682	0.385	0.814	0.686	0.874	0.639
11	1.892	1.580	0.367	0.781	0.626	0.892	0.835	0.954	0.781
12	1.918	1.715	0.368	0.788	0.634	0.844	0.894	0.900	0.839

TABLE 2. Chi-square Test Result.

		Chi-Square value					Probability of belonging to same distribution				
Month	Degrees of Freedom	Exponential	Weibull	Beta-P	Gamma Moments Estimator	Gamma Maximum Likelihood	Exponential	Weibull	Beta-P	Gamma Moments Estimator	Gamma Maximum Likelihood
						Daily Precipitation Amounts					
1	41	46	23.31	38.84	19.81	38.47	0.27	0.99	0.57	1	0.58
2	37	49.79	34.85	44.35	31.25	34.08	0.08	0.57	0.19	0.73	0.61
3	28	36.7	28.94	43	36.85	36.66	0.13	0.42	0.03	0.12	0.13
4	26	33.62	48.34	38.2	39.64	35.59	0.14	0	0.06	0.04	0.1
5	24	29	17.63	25.8	25.55	33.81	0.22	0.82	0.36	0.38	0.09
6	20	28.6	19.3	26.81	20.87	25.4	0.1	0.5	0.14	0.4	0.19
7	13	13.9	13.16	14.81	11.05	15.64	0.38	0.44	0.32	0.61	0.27
8	17	17.75	13.01	20.97	17.75	18.88	0.4	0.74	0.23	0.4	0.34
9	21	38.29	25.03	28.58	27.61	34.12	0.01	0.25	0.12	0.15	0.04
10	31	43.19	26.75	36.08	28.18	32.93	0.07	0.68	0.24	0.61	0.37
11	51	41.77	45.08	33.23	37.95	28.7	0.82	0.71	0.97	0.91	1
12	51	37.68	37.65	52.51	31.48	37.54	0.92	0.92	0.42	0.99	0.92
mean		34.69	27.76	33.6	27.33	30.99	0.3	0.59	0.3	0.53	0.38

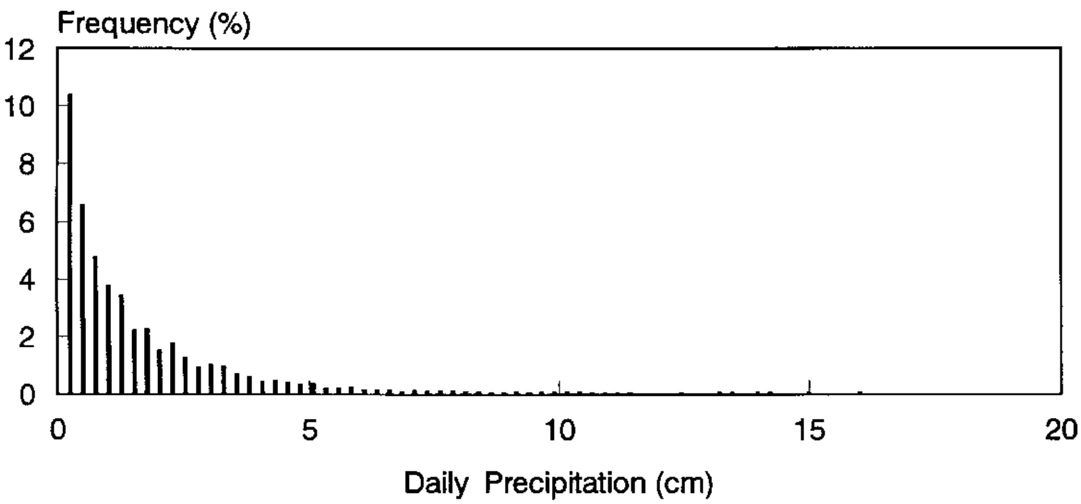

Figure 2. Observed daily precipitation distribution at HJA.

amount of precipitation being larger (about 0.8 inch) than that in the months from April to September (less than 0.5 inch), as shown in Figure 3a. The variance of daily precipitation was also higher in the wet season (October through March).

The conditional probabilities of wet day precipitation (Figure 3b) demonstrate the persistence of daily precipitation events. In the wet season, a wet day is more likely to be followed by a wet day, while in the dry season (April through September), the probability of a dry day following a dry is much higher than a wet day following a dry day. The first- order Markov chain model simulated 8,231 dry days (about 55.0% of total days), which is very close to the actually observed 8,255 dry days (about 55.1% of total days). A comparison of the observed and simulated wet spell frequency distribution is shown in Figure 4. This

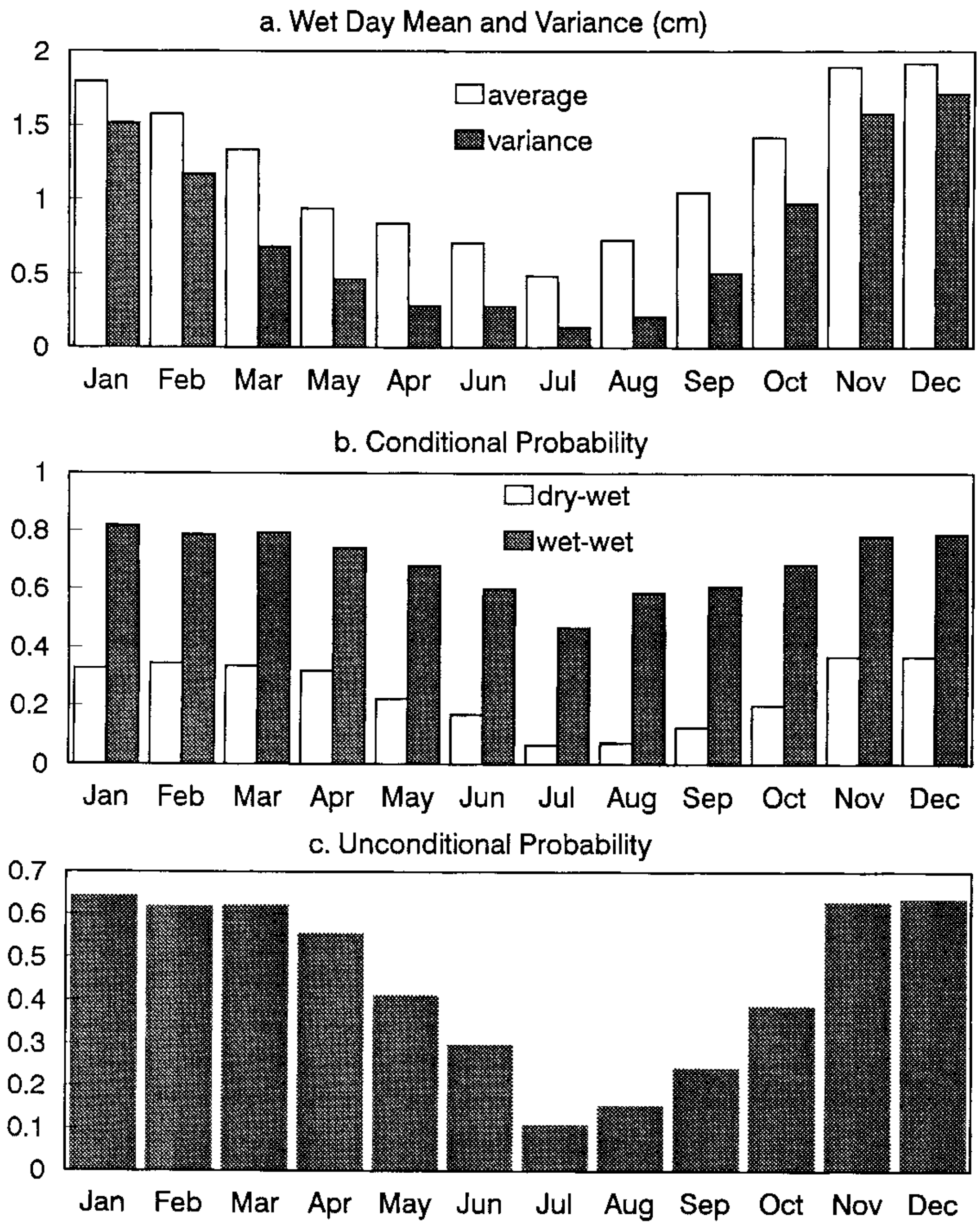

Figure 3. Model parameters for H. J. Andrews Experimental Forest.

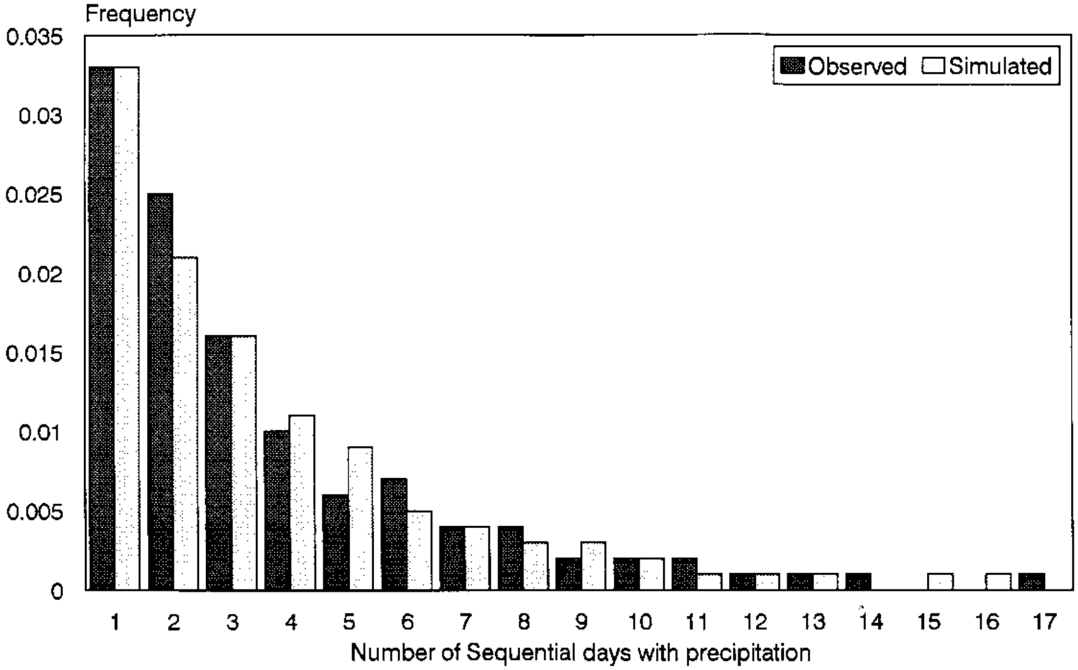

Figure 4. Markov chain performance. The true probability of a dry day observed is 55.1% and for simulated is 55.0%.

shows a close comparison of the occurrence of simulated wet days with the observed data.

Daily precipitation distributions of observed and model simulated data with different models are shown in Figure 5a. The results generated by all models closely approximate the observed data. The monthly distribution of precipitation amounts obtained by summing the daily precipitation amounts generated with each model is also shown (Figure 5b). Visually, for daily and monthly precipitation amounts, all the above models generally represent the observed data well.

To compare model performance, the summary of Chi-square test results is given in Table 2. Model performance in terms of Chi-square probabilities of different models for daily and monthly precipitation amounts is shown in Figure 6. For simulation of daily amounts, the two parameter gamma and calibrated Weibull (with higher mean probability of about 0.52 and lower Chi-square of about 29) appear better than exponential and Beta-P distributions. However, none of the above models was good at simulating daily amounts in March and April. It appears that the models tend to break down during the transition from a wet to a dry season. This may be due to the changes in the underlying process of precipitation mechanism during the transition period. For simulating monthly amounts, all the above models in general appear to perform well. The 2-parameter gamma, with moments estimator showed better results than the maximum likelihood estimator.

The distribution of daily maximum precipitation obtained from each model are compared with the observed data in Figure 7. The gamma with maximum likelihood estimator and exponential appear closer to the observed data. These results suggest that the two parameter gamma with moment estimator and calibrated Weibull should be acceptable for generating daily rainfall data for general hydrologic, ecologic, and water yield studies at the HJAEF. However, the single parameter calibrated Weibull distribution is easy to apply and equally good. The gamma distribution with maximum likelihood estimator may be preferred for conducting hydrologic studies that are sensitive to daily extreme precipitation amounts.

Our results are based on the data from the HJAEF. These results may be applicable to other areas in the western Cascades of Oregon which have similar climate. A further study is in progress to conduct this analysis for a number of

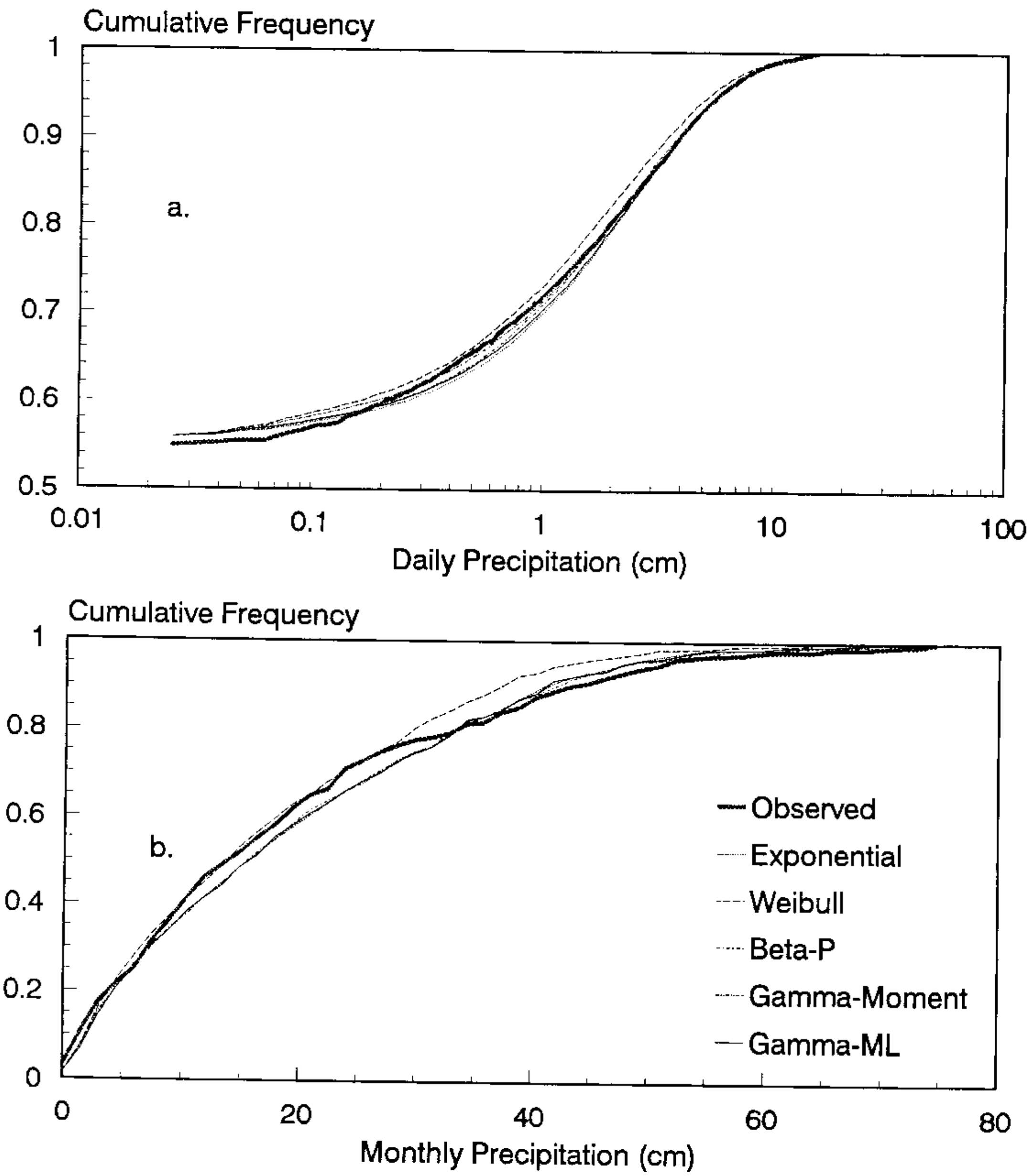

Figure 5. Comparison of observed daily and monthly amounts with models. a). Cumulative frequency for daily amount; b). cumulative frequency for monthly amount.

precipitation sites in the western Oregon, and examine the general applicability of these results to regional studies.

Conclusion

A daily precipitation model has been presented for generating daily precipitation for the H. J. Andrews Experimental Forest, based on a two-state Markov chain governing the occurrence and a theoretical distribution governing the amounts of precipitation. The exponential, calibrated Weibull, calibrated Beta-P, and two 2-parameters gamma distributions are compared in generating precipitation amounts. The Markov chain model performed well at simulating daily precipitation

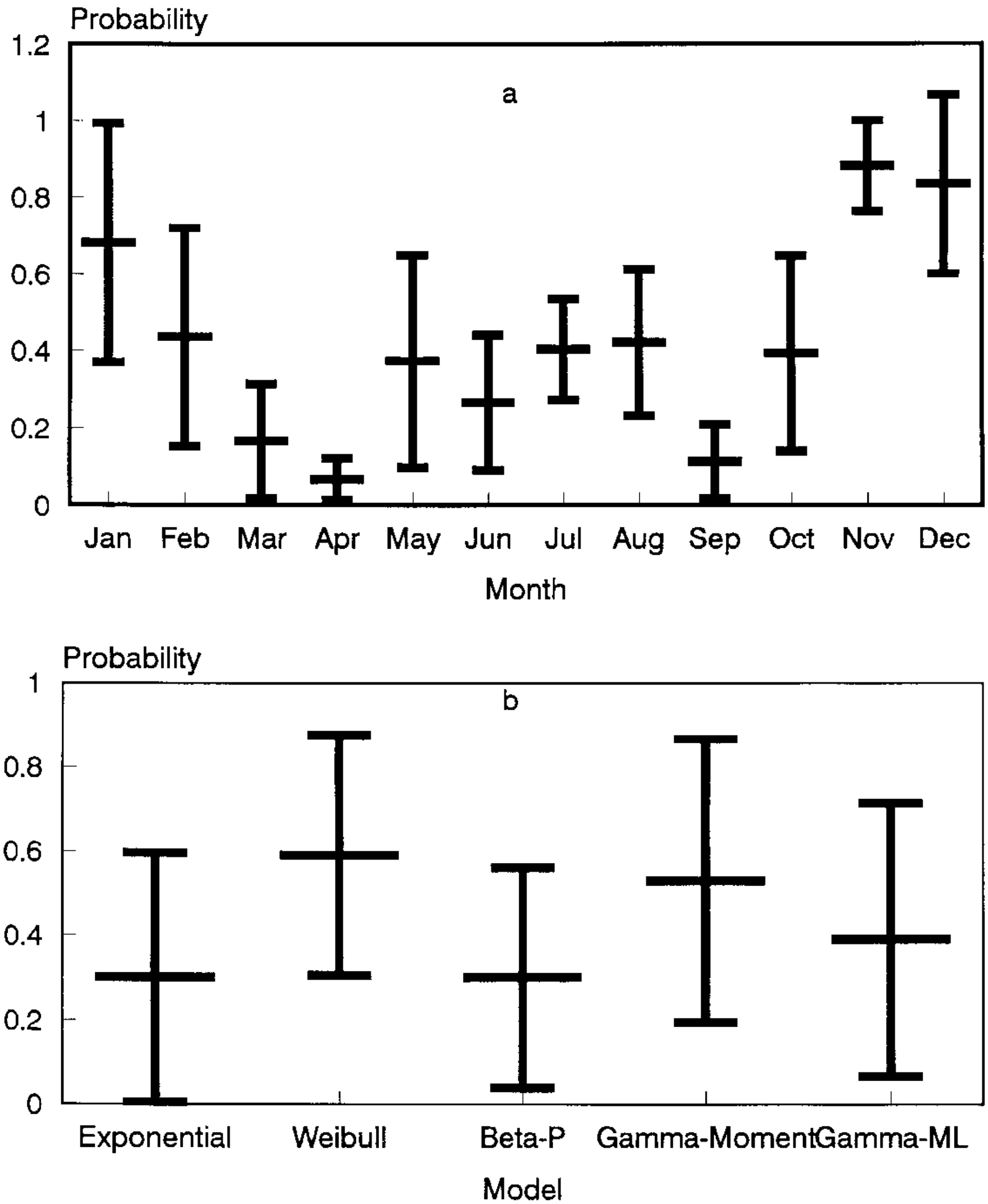

Figure 6. Model performance. a). Comparison of behavior of all 5 models combined by month; b). total probability by model for all months combined. Note mean and one standard deviation plotted.

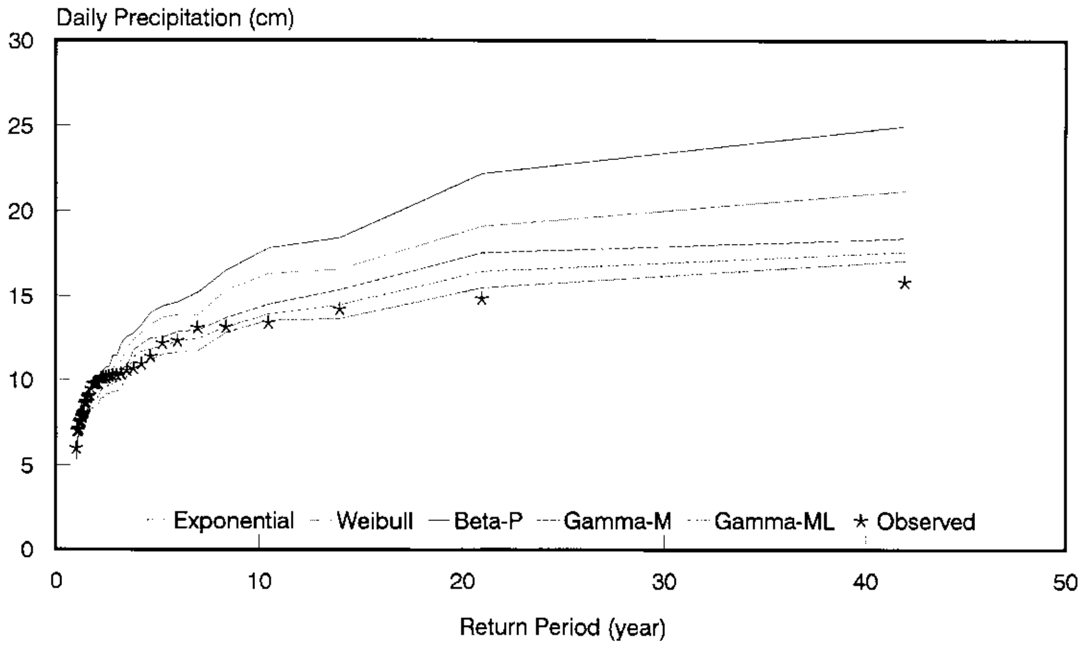

Figure 7. Comparison shows that gamma with maximum likelihood estimator and exponential appear better in generating daily extreme precipitation.

occurrences. The results also suggest that the calibrated Weibull is appropriate for generating daily precipitation amounts for its simplicity, goodness-of-fit and reversibility. The 2-parameters gamma may be preferred if the accuracy of daily amounts as well as daily extreme values are also important in a given study.

Acknowledgements

This study was funded partially by a National Science Foundation grant for the H. J. Andrews Experimental Forest, Long-Term Ecological Research Program (BSR9011663) and the US Forest Service Global Change Research Program. The daily precipitation data were provided by Mr. Don Henshaw of Forest Science Laboratory, USDA Forest Service, Corvallis, Oregon. The authors wish to thank Dr. John Selker, Department of Bioresources Engineering, Oregon State University for his many useful suggestions and comments on an early draft of this manuscript. Thanks also to two anonymous reviewers who provided helpful comments.

Literature Cited

Bailey, N. T. J., 1964. The elements of stochastic processes. John Wiley, New York, pp39.

Bierlmaier, F. A. and Art McKee, 1989. Climatic summaries and documentation for the primary meteorological station. H. J. Andrews Experimental Forest. 1972-1984, USDA Forest service. Pacific Northwest Research Station, Portland, OR. General Technical Report. PNW-GTR-223, pp26.

Buishand, T. A., 1978. Some remarks on the use of daily rainfall models. Journal of Hydrology. 36:295-308.

Chin, E. H., 1977. Modeling daily precipitation occurrence process with Markov chain. Water Resour. Res. 13(6): 949-956.

Choi, S. C. and R. Wette, 1969. Maximum likelihood estimation of the parameters of the Gamma distribution and their bias. Technometrics. 11(4):683-690.

Devore, Jay. L., 1987. Probability and statistics for engineering and the sciences. Brooks/Cole Publishing company, Monterey, California, pp672.

Gabriel, R. and J. Neuman. 1962. A Markov chain model for daily rainfall occurrence at Tel Aviv, Israel. Q. J. R. Meteorol. Soc. 88:90-95.

Greenwood, Anther J. and David Durand, 1960. Aids for fitting the Gamma distribution by maximum likelihood. Technometrics 2:55

Haan, C. T., 1977. Statistic methods in hydrology. The Iowa State University Press, Ames, Iowa. pp378.

Haan, C. T., D. M. Allen and J. O. Street, 1977. A Markov chain model of daily rainfall. Water Resour. Res. 12(3):443-449.

Mielke, Paul W. Jr. and Earl S. Johnson, 1974. Some generalized distributions of the second kind having desirable application features in hydrology and meteorology. Water Resour. Res. 10(2):223-226.

Pickering, N.B., J.R. Stedinger and D. A. Haith. 1988. Weather input for nonpoint source pollution models. J. Irrig. Drain. Eng. 114(4):674-690.

Press, William H., Saul A. Teukolsky, William T. Vetterling and Brian P. Flannery, 1992. Numerical Recipe in C. Cambridge University Press, NY 10011-4211, pp 994.

Richardson, C. W., 1981. Stochastic generation of daily precipitation, temperature, and solar radiation. Water Resour. Res. 17(1):182-190.

Richardson, C. W., 1982. A comparison of three distributions for the generation of daily rainfall amounts. In: Statistical analysis of rainfall and runoff, pp. 67-78, V. P. Singh (ed.). Water Resources Publications, 700 pp.

Richardson, C. W., and D. A. Wright, 1984. WGEN:a model for generating daily weather variables, U.S. Dep. Agric., ARS-8, 83p.

Rodriguez, R. N., 1977. A guide to the Burr type XII distributions. Biometrika. 64:129-134.

Received 25 February 1995
Accepted for publication 27 June 1995

Roldan,Jose and D. A. Woolhiser, 1982. Stochastic daily precipitation models: 1. A comparison of occurrence processes. Water Resour. Res. 18(5):1451-1459.

Selker, J. S. and D. A. Haith, 1990. Development and testing of single-parameter precipitation distributions. Water Resour. Res. 26(11):2733-2740.

Smith, R. E., and H. A. Schreiber, 1974. Point processes of seasonal thunderstorm rainfall, 2. Rainfall depth probabilities. Water Resour. Res. 10(3):418-423.

Thom, H.C.S., 1958. A note on the Gamma distribution. Monthly Weather Rev. 86(4):117-122.

Todorovic, P. and D. A. Woolhiser. 1974. Stochastic model of daily rainfall, Proceedings of the Symposium on Statistical Hydrology. U.S. Dep. Agric. Misc. Publ. 1275, 232- 246.

Todorovic, P. and D. A. Woolhiser, 1975. A stochastic model of n-day precipitation. J. Appl. Meteorol. 14(1):17-24.

Woolhiser, D. A., E. Rove and P. Todorovic, 1973. Temporal and spatial variation of parameters for the distribution of n-day precipitation. In: Flood and Droughts: proceedings of the Second International Symposium on Hydrology, pp. 605-614, Water Resources Publications, Ft. Collins, Colorado.

Woolhiser, D. A. and Jose Roldan, 1982. Stochastic daily precipitation models: 2. A comparison of distribution amounts. Water Resour. Res. 18(5):1461-1468.