

NOTES ON THE CALCULATION OF THE VOLUME OF FLOW

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DEFINITION:

Equation is a single relationship with fixed set of constants for certain range of stage heights

Break point is the maximum stage height where the defined equation can be used.

Rating curve is a set of equations that cover all possible stage heights.

ONE EQUATION VOLUME BETWEEN TWO POINTS

Assume between time t_1 and t_2 , the stage height moves from h_1 to h_2 in a linear relationship and the equation

$$\ln(Q) = a + b \ln h + c (\ln h)^2 + d (\ln h)^3$$

can be used for any point within this time or stage domain, with a, b, c, d are fixed constants for this time.

Then the total volume of water produced during this period is

$$V = \int_{t_1}^{t_2} Q dt = \frac{1}{R} \int_{h_1}^{h_2} Q dh$$

with $R = dh/dt = \text{constant}$.

Several situations are considered:

1) $R = 0$

$$V = Q(t_2 - t_1)$$

2) Other

1. $C = d = 0$;

A) If $b = -1$,

if either h_1 or h_2 is zero

$$V = (Q_1 + Q_2) * (t_2 - t_1) / 2$$

else

$$V = \frac{1}{R} \int_{h_1}^{h_2} Q dh = \frac{e^a}{R} \int_{h_1}^{h_2} \frac{1}{h} dh = \frac{e^a}{R} \ln\left(\frac{h}{h_1}\right)$$

B) Else

$$V = \frac{e^a}{R(b+1)} (h_2^{b+1} - h_1^{b+1})$$

2. C or d is not zero
(The Andrews flumes no longer use equations of this type- Dec 2002)

Virtually, there is no analytical solution, and a numerical solution has to be used. Since the integral may be undefined at the low boundary (logh not defined when $h = 0$), the midpoint method is used. Please see numerical textbook for details.

VOLUME BETWEEN ANY TWO POINTS

Volume calculation between any two points is identified and partitioned using the following:

- If the two points can be calculated using one equation, which means that any point in this period can be calculated using one equation, then do above.
- If the two points cross equation boundaries, then separate by breakpoints, do above for each sub-period and sum all those results from different equations.
- If the two points cross rating curves, separate by the breakpoint for rating curve and sum results together.