NOTES ON THE CALCULATION OF THE VOLUME OF FLOW
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DEFINITION:
Equation is a single relationship with fixed set of constants for certain range of stage heights
Break point is the maximum stage height where the defined equation can be used.
Rating curve is a set of equations that cover all possible stage heights.

ONE EQUATION VOLUME BETWEEN TWO POINTS
Assume between time t₁ and t₂, the stage height moves from h₁ to h₂ in a linear relationship and the equation
\[ \ln(Q) = a + b \ln h + c(\ln h)^2 + d(\ln h)^3 \]
can be used for any point within this time or stage domain, with a, b, c, d are fixed constants for this time.
Then the total volume of water produced during this period is
\[ V = \int_{t_1}^{t_2} Q \, dt = \frac{1}{R} \int_{h_1}^{h_2} Q \, dh \]
with R = dh/dt = constant.

Several situations are considered:
1) \( R = 0 \)
   \[ V = Q(t_2 - t_1) \]
2) Other
   1. C = d = 0;
      A) If b = -1,
         if either h₁ or h₂ is zero
         \[ V = (Q_1 + Q_2) \frac{1}{2}(t_2 - t_1) \]
         else
         \[ V = \frac{1}{R} \int_{h_1}^{h_2} Q \, dh = \frac{e^a}{R} \int_{h_1}^{h_2} 1 \, dh = \frac{e^a}{R} \ln \left( \frac{h_2}{h_1} \right) \]
      B) Else
         \[ V = \frac{e^a}{R(b + 1)} (h_2^{b+1} - h_1^{b+1}) \]
2. C or d is not zero  
   (The Andrews flumes no longer use equations of this type- Dec 2002)

Virtually, there is no analytical solution, and a numerical solution has to be used. Since the integral may be undefined at the low boundary (logh not defined when h = 0), the midpoint method is used. Please see numerical textbook for details.

VOLUME BETWEEN ANY TWO POINTS

Volume calculation between any two points is identified and partitioned using the following:

- If the two points can be calculated using one equation, which means that any point in this period can be calculated using one equation, then do above.
- If the two points cross equation boundaries, then separate by breakpoints, do above for each sub-period and sum all those results from different equations.
- If the two points cross rating curves, separate by the breakpoint for rating curve and sum results together.