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ON CONSTRUCTING A HIERARCHICAL MODEL IN THE FLEX PARADIGM, ILLUSTRATED BY STRUCTURAL ASPECTS OF A HYDROLOGY MODEL†

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A general strategy for constructing hierarchical models of complex systems is introduced and compared with a pure analytic and pure synthetic approach. The strategy is a formalization of standard scientific procedures for gaining understanding of systems. The strategy is stated and demonstrated in terms of the FLEX modelling approach and paradigm. Some details of the FLEX hierarchical structure, especially coupling specifics, are elaborated.

The actual demonstration of the strategy spans two papers. The first covers the evolutionary development and elaboration of a single level hydrology model, including the elaboration of considerable mechanistic detail.[8] The present paper completes the example by exhibiting how the structure implicit in the elaborated single level model is explicitly identified within the REFLEX hierarchical structure.

INDEX TERMS: Hierarchical model, FLEX modelling, hydrology model, REFLEX structure

INTRODUCTION

Current mathematical models of ecosystems are so complex and large that it is extremely difficult to understand how the model behaves, much less to conceptually master the details of the couplings and interactions. Methods need to be developed to reduce the overall complexity so that model building and the study of model behavior will be made easier, but without sacrificing the size and scope of the systems studied, nor the necessary detail required by the questions posed. Hierarchical model forms appear to be useful in reducing complexity. Our use of hierarchical forms is strongly reinforced by the emergence of a general theory which holds that hierarchical structures are the natural order of complex systems, it being recognized that such structures are fundamentally more stable and realizable[1,2] and theoretically more realistic.[3,4] In accordance with this emerging view, our modelling efforts have been directed toward development of hierarchic models of ecosystems. Many details of this approach have been reported.[5,6,7,8]

This paper reviews some aspects of the process of building a hierarchical model, and illustrates the process by a hydrology model of a unit watershed. Use of this model in parameter estimation, study of model behavior and subsystem elaboration are briefly treated. The model described is closely similar to the final version of the model reported in the paper by Overton and White.[8] The model was developed first in the single level mode for non-hierarchic systems, which we call FLEX. During development, subsystems were identified and it became apparent that the various behaviors were isolated, or nearly isolated, on these subsystems. The hierarchic model form, here described, was then developed to express these properties in the multi-level mode for hierarchic systems, which we call REFLEX.

The subsystems of the watershed ecosystem were developed during a series of meetings, referred to as Modelling Round One of the Central Modelling Group and subject colleagues of the Coniferous Forest Biome, US/IBP, in Winter and Spring 1971. These structures are reported in part by Overton.[5] Our discussions were initially oriented to compartment models, but compart-

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containing many parts. Our FLEX/REFLEX module is a realization of the holon concept in the form of an explication and elaboration of Klir's definitions. This structure is viewed symbolically in Figure 1(b). Reference is made to White and Overton[7] for details of the module structure.

We implement the conceptualization of the behavioral model form by a *box and arrow* diagram, representing variables coupled by processes. The arrow technically represents a physical or information flow *and* the process which is responsible for that flow, but we often introduce small circles to explicitly identify processes. Then a *circle and diamond* diagram, representing subsystems coupled by variables, stands for the U-C structure. This is illustrated in Figure 2.

The conceptual U-C structure must be translated according to the algorithm which we chose for our computer processor, Figure1(c). Specifically, the system or subsystem which is being modelled is represented by a *module*, which may be either a FLEX module, or a REFLEX module. If it is a FLEX, then it contains a representation of the behavioral structure, including all process functions for updating the variables. If it is REFLEX, then it contains some functions required to update variables and to complete couplings among the subsystems, but sends information to those subsystems and receives variable updatings from them. Temporal resolution may be finer in the subsystem, if desired. For example, a system operating at one week resolution may have its subsystems operating weekly, daily, hourly, or otherwise in any integral fraction of a week.

A REFLEX module is also called the ghost system (after Koestler[3]) to distinguish it from the proper subsystems of the conceptual structure. It not only provides the capacity of integration and control over the subsystems, allowing the potential of whole system behavior which transcends the behavior of the individual subsystems, but it also provides the capability of parallel processing. That is, if subsystem one takes something from subsystem two, according to a relation depending on the state of each subsystem, then the necessary information for that transaction can be sent to ghost, the "decision" calculation made there and communicated to each subsystem in the same time period. Alternatively, it is possible either to have a lag in updating, or to provide for strict sequential processing with immediate sequential communication among subsystems within a cycle. The distinction between parallel and sequential forms will be illustrated with the model reported here.

State variables differ from coupling variables in a subtle, often superficial way, but the distinction is essential to the conceptual rigor of the paradigm. State variables are not identified in Klir's theory; we have reinstituted the term, but in our

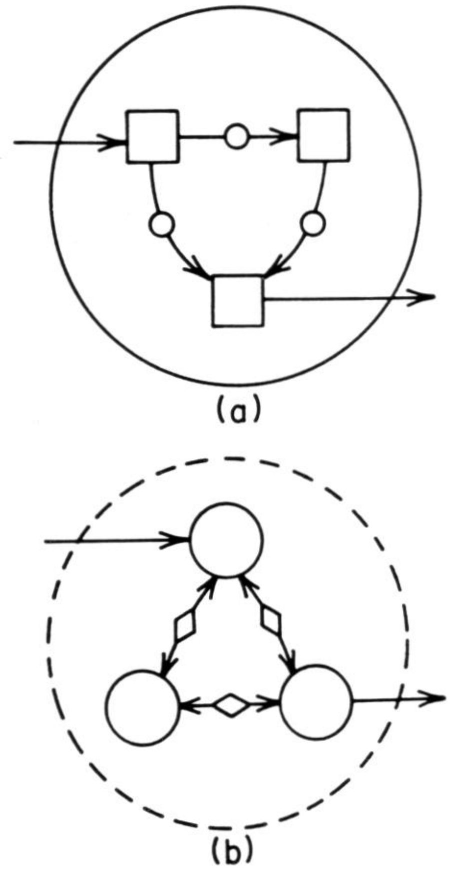


FIGURE 2 The dual model representation allows alternate emphasis on process and variable. The conceptual distinction between coupling variables and state variables is recognized by the use of diamonds and boxes, respectively.

a) The Behavioral mode is depicted by box and arrow diagram emphasizing variables, with connecting small circles representing processes. A Forrester Diagram is often useful for this mode, as is the distinction between "physical" flows and "information" flows.

b) A circle and diamond diagram depicts the Universe-Coupling mode. Circles represent processes or subsystems, which can in turn be represented by either mode.

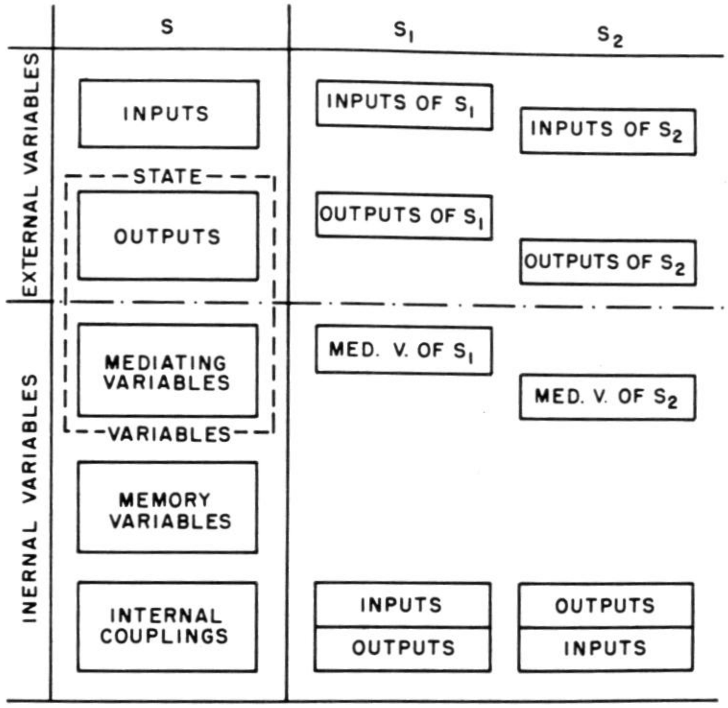


FIGURE 3 Schematic representation of variable definition in FLEX and elaboration under decomposition of a whole system into two sybsystems. New, internal variables of S must be defined to accommodate coupling variables between the subsystems. It is not necessary for all outputs of S to be identified as explicit outputs of S_1 or S_2.

a) Variable definition and relationship in FLEX mode.

b) Elaboration of variables under decomposition, i.e., in REFLEX mode.

of the whole system and are not filtered or integrated by total system behavior, then these outputs may be sent directly to file storage without passing through ghost. This also permits the capacity of retaining the higher output resolution of the subsystem.

Mechanically, if a particular state variable or process (**g**) function value in subsystem one is desired as an input by subsystem two, then that variable or function is designated an output of subsystem one, a state variable, x_i, of ghost is defined to correspond to said output, and that x_i is specified to be an input by subsystem two.

In order to accommodate passage of coupling variable values, a discrete time structure is used. The subsystems are uncoupled from each other during any time step k and the value of $x_i(k)$ in ghost is used as the coupling variable value during this time. (Remember, inputs, $\mathbf{z}(k)$, and altered coupling variable values, $\mathbf{g}(k)$, in ghost are available as well.) This presents no problem for parallel processing since the "current" information of all subsystems is available and subsystems may be processed in any order. In sequential processing, however, the order of subsystem processing is important since the input to a subsystem may depend on the output of a subsystem previously processed during the same time step. These values are made available using the vector $\mathbf{x}s(k)$ where

$$\mathbf{x}s(k)=\mathbf{x}(k+1). \tag{1}$$

The use of a discrete time coupling structure poses no problems for discrete time models, but adjustments must be made for continuous time models. In essence these models must be "piecewise" integrated, that is, integrated over the discrete time interval, in isolation from each other.[6] This requires that the system be decomposed into subsystems that are relatively isolated or uncoupled from each other so that their interaction will be at a coarser time resolution than their internal dynamics.

Normally, when a system is decomposed, its subsystems operate at a finer time resolution and the ghost system must couple the subsystem time resolution to the time resolution of the whole system level. Expansion of the time resolution is accomplished within ghost by specifying an integer q for the number of subsystem time steps to every whole system time step. To satisfy this additional cycling, an additional time variable is introduced. This results in the representation of an x variable as $x(k)$ in the "upper" part of a module, and representation of the same variable as $x(k, k')$ in the "lower" part. Then,

$$x(k)=x(k, 0) \tag{2}$$

$$x(k, k'+1)=x(k, k')+\Delta(k, k'), \quad k'=0, 1, \ldots, q-1 \tag{3}$$

$$x(k+1)=x(k, q) \tag{4}$$

Within the FLEX or terminal nodule the time resolution may be expanded also, again using q, so that subsystems at the same level, while cycling the same number of times externally, may cycle more or less frequently internally and thus conceptually represent different time resolutions. Additional details on algorithm processing may be found in White and Overton.[7]

STRATEGIES FOR CONSTRUCTING A HIERARCHICAL MODEL

Against this background of consideration of the manner in which we postulate hierarchical structures are coupled, we next turn to strategies for developing such structure. Our present view of this topic is obviously very primitive; our experience is limited, and we have little support from experience of others. Yet it is an essential aspect of the developing ideas which we are reporting, so even a primitive treatment is desirable.

Recognizing that this activity is essentially one of model building, within a particular class of models, we must acknowledge that a strong subjective element will dominate the process. Modelling is an art, and will always be an art, but any art demands mastery of relevant techniques. We will attempt to subjectively verbalize our present ideas concerning the techniques of constructing hierarchical models.

Several distinct strategies can be identified. First, one can start with a behavioral model of the whole system, which requires identification of the essential behaviors of interest, then decompose the holistic behavior, and construct submodels for each sub-behavior. Second, one can start with a single level mechanistic model (i.e., one in which an attempt has been made to construct the internal mechanisms for how the system works) and then partition the structure into sub-structures such that whole system sub-behaviors are isolated on the sub-structures. Third, one can begin with a view of all of the parts, construct a model for each, and then assemble the parts into a whole, synthesizing whole system behavior from the sub-behavior of the parts. Fourth, the model objectives can be decomposed, sub-objectives related either to sub-behaviors or sub-structures, the parts built and then assembled into the whole. The first two strategies are analytic in nature, the third synthetic, and the fourth a mixture with analysis preceding synthesis.

In the following discussion, we shall attempt to establish the position that any specific modelling effort will incorporate elements of each strategy and that difficulties can be identified in each which relate to fundamental modelling problems. The first strategy, which can be identified as an analytic/decomposition strategy, requires the identification of holistic properties and behaviors. Each of these is then modelled as by our FLEX mode. The identification of subsystems in the form of structures *explaining* the individual behaviors is such an abstract concept that it is difficult to embrace. The difficulty is compounded by the fact that seldom (perhaps never) do we approach a problem in total ignorance of how the system works, so that it is difficult to conceive of an example in which some conceptual sub-structures do not exist in our minds, even if we have forced ourselves to exclude them from the behavioral model. To this point, we observe that the process of constructing subsystems has already been informally engaged; the first strategy is rightly considered a formalization of processes which characteristically accompany the study of systems phenomena. Thus, in implementation of the first strategy, we would not attempt to rely solely on techniques of analyzing and decomposing behavior, but rather will also allow representation of the perceived structure of the system which has evolved in its less formal study. To do otherwise would be to deny existing knowledge.

The second strategy may also be identified as an analytic/decomposition strategy, except that in this case we start with a structural (mechanistic) representation of model behavior and attempt to delineate sub-structures which are responsible for different behaviors, which are then properly considered the sub-behaviors. Again, we cannot engage the problem in total isolation of existing knowledge. Usually much of the mechanistic structure is already informally identified with sub-behaviors (often conceptualized as sub-processes) as these formed the criteria by which the structure was included in the first place.

The third strategy may be identified as a synthetic/composition strategy, and it formally requires that subsystem models (in the FLEX mode) be coupled together into models for higher level systems in the REFLEX mode. Behaviors of the whole then "emerge" from composition of the subsystem behaviors. The difficulty here is that the manner of coupling determines the composition, or holistic behavior. What observations, other than direct observation of whole system behavior, will give clues to the nature of these couplings? Again, it is necessary to recognize that the perception of the system as a coupled collection of subsystems requires some concept of the nature of the whole, and that this "nature" essentially constitutes the holistic behavior.

The fourth strategy may be identified as an analytic/composition strategy. The analysis is done in the objective space while the composition is done in behavioral space utilizing holistic or mechanistic structures. When these structures are composed, the total behavior, as defined by the total model objective, should result. This approach suffers from the problems of all previous strategies since the objectives are formulated partly in the light of existing knowledge about what may be studied and the objectives will usually be stated in such a way as to require that certain known behaviors and structures be included in the model. And the total model objective usually centers on an understanding of whole system behavior and how decomposing the objective interrelates with holistic behavior.

Thus in all four strategies we see that other considerations enter subjectively time and time again, and that these considerations rest largely on our partial informal knowledge of the system modelled.

An insight into problems of studying natural ecosystems is provided by contrast of the first and third strategy. Systems which are readily perceived as objects are readily studied as wholes. With the first strategy, characterized as analytic/decomposition, questions pertaining to the nature of the *behavior* of the objects are answered. These are the *WHAT* questions, such as, what is the nature of the system? Then attention turns to *mechanistic* questions; these are the *HOW* questions, such as, how does the system work? These questions look downward in the hierarchy, requiring decomposition of the system into its parts, and identification of new structures responsible for the decomposed behavior. This has been the dominant activity of "normal" science for several centuries and the analytic approach is well understood.

To look upward in the hierarchy is to ask *WHY* questions, which are strongly teleological, mystical and supposedly forbidden. With the third strategy, it is only by the intuitive perception of an object at the next higher level (i.e., by answering the *WHAT* questions at that higher level) that one can establish the position to address phenomena in the *HOW* or *mechanistic* or *reduction* mode, and so eliminate the elements of mysticism.

The intuitive perception of higher level objects or systems is a creative activity, and the process of building an explicit model of a system in the synthetic mode requires first the intuitive identification of that system as an object, in terms of its holistic behavior. Then the construction of a structural model using submodels must include analytic considerations, since the parts must be *designed* in such a manner that the submodels will couple together to yield the desired behavior of the whole system.

Thus, we find it is necessary to use a mixture of the basic strategies. The procedure we employ may be generalized something as follows. A holistic model in the behavioral (FLEX) mode is constructed. Behavior is represented by simple mathematical forms which comprise the model structure. Internal model structure is elaborated until sufficiently precise behavior is obtained, the model evolving toward the mechanistic, but still in the FLEX mode. In the process of refining the structure and studying parameter values, hierarchical structure begins to be apparent to the modeller, adding to that structure which was earlier expressed using the conventional knowledge of the system being modelled. At some point of increased complexity it will become advantageous to impose the formal structure of the REFLEX mode, making *explicit* the hierarchical structure which has been *implicit* in the mechanistic FLEX model. This process creates subsystems within which the mechanistic structure has been reduced, and perhaps eliminated. The subsystem representations are now behavioral, but the procedure may be again applied to any subsystem, yielding a multi-level hierarchy.

It was in this general manner that we developed the hierarchical hydrology model, which is summarized in the next section of this paper. A series of models was developed, each new member of the series being designed to improve the behavior in some explicit manner. In the process we become aware that certain behaviors were dependent on recognizable subsystems. It was also apparent that the physical structure conventionally considered in hydrologic processes was quite well associated with the recognized behavioral structure. In only a few cases did we identify the need for structures which apparently did not already exist in the hydrology paradigm. This strengthens our position that modelling of this sort is rightly considered a formalization of the process of normal science.

THE HIERARCHICAL HYDROLOGY MODEL

Background

The model here presented evolved through a series of some 15 versions in the process reported by Overton and White.[8] The model form which terminated that activity is the base for the example used in the current paper. Some minor changes have since been implemented, to accommodate recognized needs of a Watershed Ecosystem Model, of which this hydrology model was designed to be a sub-model. At the time of identification of the Watershed Ecosystem Model structure (Modelling Round 1, Coniferous Forest Biome, US/IBP, Spring, 1971), the couplings between the major subsystems (hydrologic, primary production, consumer/decomposition, and nutrient interchange and uptake) were tentatively established. For example, it was specified that the couplings between *primary production* and *hydrology* were *soil water profile* and *transpiration.* In developing the hydrology model, however, it was necessary to incorporate a component to determine transpiration, this component effectively representing the function of the primary production subsystem which is coupled to the hydrology subsystem. Also included are essential meteorological processes such as evaporative demand, which one might question as belonging to "hydrology", and which could easily be externalized as coupled system structure.

The objective of the hydrology modelling activity was simulation of coarse dynamic behavior as measured by several criteria: annual budgets of transpiration, evaporation and streamflow; timing and magnitude of hydrograph peaks; and two statistical objective functions of the daily hydrograph $\Sigma(y-\hat{y})^2$ and $\Sigma(\ln y - \ln \hat{y})^2$. Hydrograph quantities were evaluated against observations, other quantities against average values for the general area. Model behavioral improvements were achieved by modifying the model structure rather than fine-tuning parameters.

In submodel construction, it is important to note that no observational data were available for subsystem couplings, which were internal variables of the single level model. This clearly points to a major difficulty in developing these models; much attention must be given to the problems of identification of internal variables which *can* be observed, and in collecting data sets which include these variables and which can be used in running and verifying the models.

The relationship of the model described (S_{11}) to the Watershed Ecosystem Model is illustrated in Figure 4. The ecosystem (S) is coupled to the environment, E. Two subsystems of S are then identified: S_1 is the terrestrial watershed ecosystem and S_2 the aquatic watershed ecosystem. Next, the terrestrial ecosystem in decomposed into four subsystems: S_{11}, hydrologic; S_{12}, primary production; S_{13}, consumer/decomposition; and S_{14}, nutrient interchange and uptake.

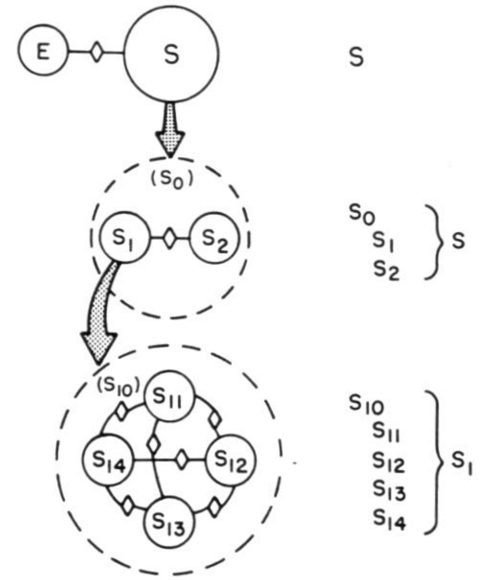


FIGURE 4 The decomposition of the watershed ecosystem into its major subsystems. Specification of the coupling variables allows the subsystems to be modelled individually in isolation.

The Single Level Model

The single level version of the hydrology model is diagrammed, using the box and arrow convention, in Figure 5. The process of developing the model will be briefly summarized here, with particular regard to the aspects which bear on identification of structure. The model began with explicit representation of snow, free canopy water, and water in two soil compartments, root zone and subsoil. The watershed of interest was physically parameterized to provide essential model parameters such as area storage capacity and field capacity for both soil compartments, and transpiration resistance point for the root zone soil compartment. All capacities were expressed in metric tons of water (m^3 or $m^3\,ha^{-1}$ depending on the usage). Other parameters, such as rate of interception by canopy and canopy storage capacity, were determined by largely subjective processes, and were secondarily subject to tuning. Still other parameters, such as rates of evaporation, percolation and interflow, were chosen primarily with the view of providing acceptable model behavior.

The process of refinement was highly subjective. We relied strongly on intuitive understanding of the effect of a structural change in proposing that change to correct a poor behavioral feature. In making structural changes, we became aware that the annual budget of evaporation was not affected at all by changes in the properties of the root zone. This is obvious, given that one is oriented to the isolation of behavior on subsystems. It follows that those structural parts which determine evaporation and transpiration are also the determinants of annual streamflow, so that the behavioral consequences of the remainder of the underground model structure were reflected primarily in the temporal distribution of the hydrograph, that is, in higher frequency (daily) responses.

But hydrograph responses are not perfectly isolated on the below ground system, either. The snow model, in particular, influences hydrograph outflow peaks to a great degree, eliminating a peak if the model calls for snow when there was no snow and yielding spurious peaks when the spurious snow melts. Evapotranspiration behaviors exert minor influences on hydrograph responses by influencing the amount of storage capacity available to be filled when an input is received but, by and large, annual behavior is separable from the high frequency responses of the daily hydrograph, and the above and below ground structures are strongly related to this separation. The existence of only two couplings (residual evaporative demand and infiltration, Figure 5) also makes this a good point for decomposition, according to the general principle of minimization of couplings between structures.

In considering the substructures within each of the two major subsystems, we can now identify the snow submodel as critical to one aspect of behavior, and the canopy to another. Below ground, we identify root zone and transpiration as related, exhibiting low frequency *and* high frequency behavior, with other processes influencing high frequency hydrograph response. Beyond these perspectives, we must be arbitrary in constructing subsystems, until we have gained more insight into the system response. To this end, we will identify subsystems for the objective of parameter tuning, and specify that four subsystems will be structured within each of the two major divisions. Structures identified for these purposes are described in Figures 6 and 7.

Identification of Subsystems

The hydrologic system is decomposed into its

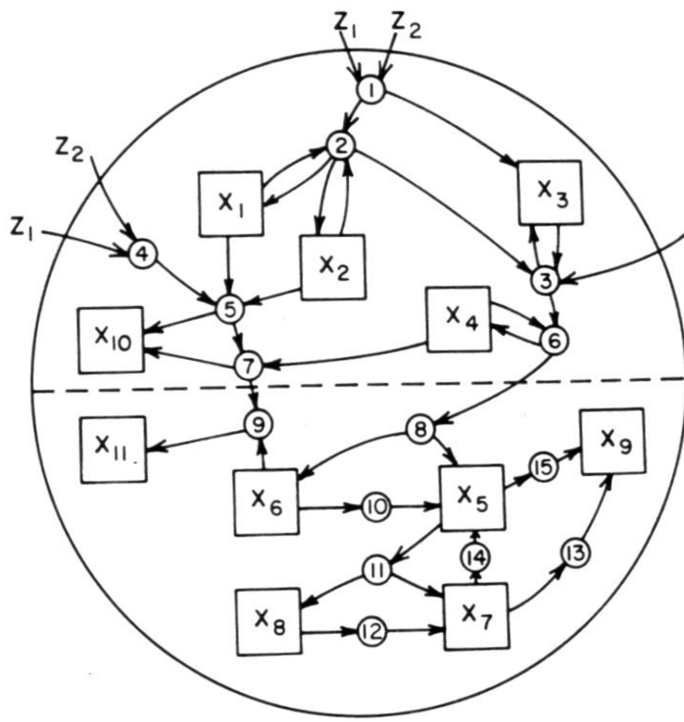


FIGURE 5 The single-level model resulting from model evolution.[8] Note that x_4 has been added.

X List (Boxes)

x_1	Foliage storage
x_2	Epiphyte storage
x_3	Snow storage
x_4	Litter layer storage
x_5	Channelized root zone storage
x_6	Non-channelized root zone storage
x_7	Channelized subsoil storage
x_9	Stream flow
x_{10}	Evaporation
x_{11}	Transpiration

Inputs

z_1	Precipitation
z_2	Air temperature

Processes (Circles)

1: Split precipitation into rain and/or snow.
2: Canopy charge, drip and thrufall.
3: Snow melt.
4: Evaporative demand calculated.
5: Canopy evaporation.
6: Litter infiltration.
7: Litter evaporation.
8: Root zone infiltration.
9: Transpiration.
10: Non-channelized root zone water lateral flow and overflow.
11: Percolation.
12: Non-channelized subsoil water lateral flow.
13: Channelized subsoil lateral flow.
14: Backflow (non-accepted percolation).
15: Channelized root zone water lateral flow.

two constituent subsystems, the above-ground subsystem, S_{111}, and the belowground subsystem, S_{112}. Each of these subsystems is in turn divided into four subsystems. The above-ground subsystem, Figure 6, is composed of: a meteorologic subsystem, S_{1111}, which determines if precipitation is in the form of rain or snow and determines potential evapotranspiration; a canopy subsystem, S_{1112}, which takes into account water stored externally in the canopy; a snow pack subsystem, S_{1113}, which determines snow pack dynamics; and a litter layer subsystem, S_{1114}, representing processes not included in our earlier forms but added here because of importance in the ecosystem model.

Elaboration of the belowground subsystem substructure is shown in Figure 7. It is composed of: a transpiration subsystem, S_{1121}, which generates the primary producers' water demand; a non-channelized root zone subsystem, S_{1122}, which handles the flow of water through the non-channelized upper soil structure (the root zone

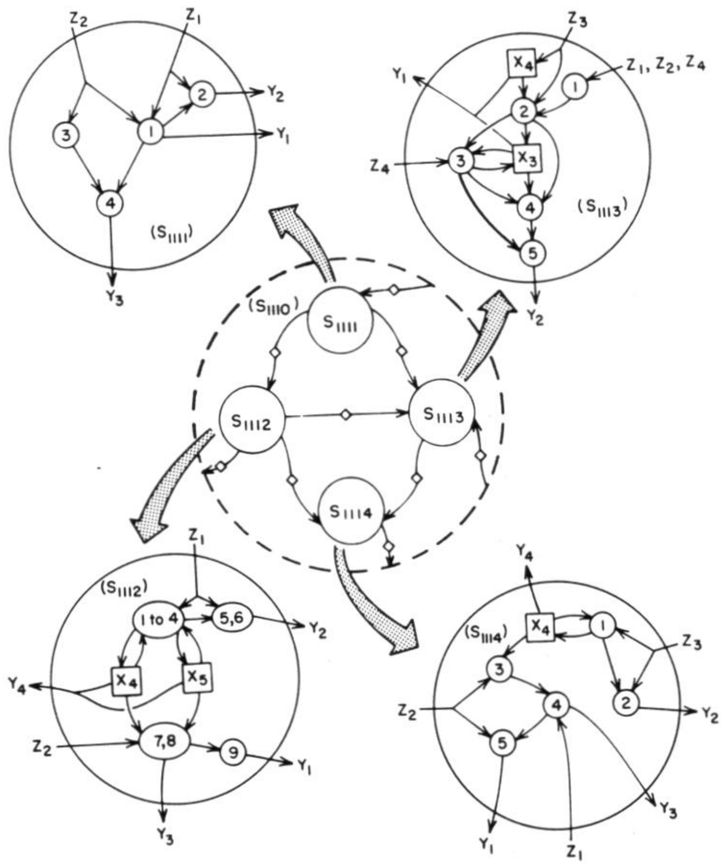


FIGURE 6 Elaboration of the aboveground subsystem into its component subsystems.

being that portion of the soil where 90% of all roots occur); a subsoil subsystem, S_{1123}, which handles percolation, interflow and 'backflow' when capacity is exceeded. The channelized root zone subsystem, S_{1124}, is conceptually awkward. It calculates streamflow from the root zone, integrating 'backflow' (nonaccepted percolation) with root zone water lateral flow. However, no lateral flow occurs in the root zone unless the subsoil is so saturated that not all percolation may be accepted. In addition, subsoil contribution to stream flow is added to any root zone water lateral flow to estimate stream flow. Conceptually speaking, this should probably be integrated at the level of the belowground subsystem. Such restructuring is straightforward and will probably be done in the future.

Subsystem S_{112} provides a good example of the convenience of sequential processing. Each subsystem of S_{112} calls variables updated by other subsystems during the same cycle, and the modeller must be sure that the sequence of processing is correctly specified. If such a sequence could not be implemented, it would be necessary to resort to the functional capacity of the ghost module to impose parallel structure on the system; values needed by two or more subsystems would be calculated in ghost prior to subsystem cycling, and communicated in arbitrary sequence to the subsystems.

Each of these subsystems and its coupling variables and internal structures will be elaborated below. These subsystems are all illustrated in the FLEXFORM summaries in the Appendix. In viewing these structures, it is necessary to bear in mind the purpose of this decomposition; the explicit purposes were: (1) to test the decomposition mechanics provided by the REFLEX

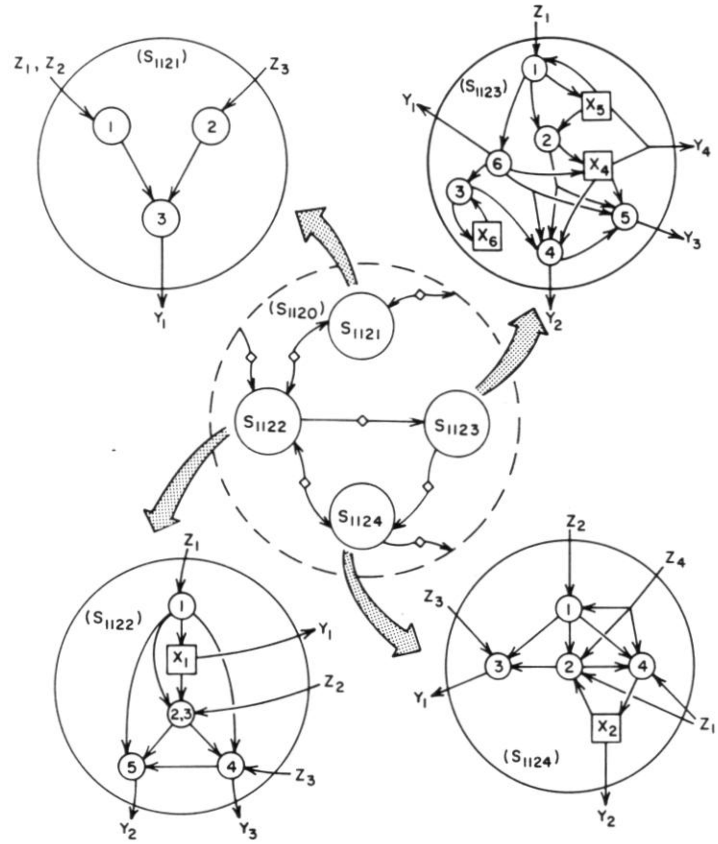


FIGURE 7· Elaboration of the belowground subsystem into its component subsystems.

model form; (2) to examine the process of sensitivity analysis in the uncoupled form; and (3) to conduct an uncoupled sensitivity analysis on this model. Several features of the decomposition are undesirable, as has been pointed out, but these features are retained in the form presented for the illustrative value. It is a relatively simple matter to restructure the subsystem in a more realistic manner, and this is clearly needed, particularly if the subsystems are to be further elaborated.

System Description

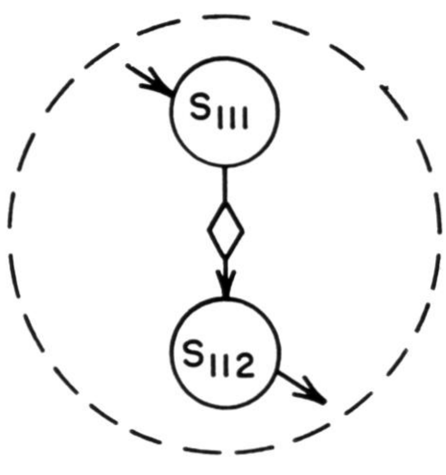
In this section we will describe the coupling variables and internal structure of the subsystems previously identified. Constant reference should be made to the FLEXFORM summaries of the Appendix, cross checking their ghost coupling lists with Figure 8. We will review these subsystems in their order of processing, which coincides with the conceptual order of water flow through the watershed. The following paragraphs are verbal descriptions of the behaviors of the subsystems.

S_{1111}, the meteorological subsystem, separates precipition into rainfall and snowfall on the basis of air temperature according to a linear function. Temperature is used to calculate potential evapotranspiration and this is modified by humidity, expressed here as a function of rainfall.

S_{1112}, the canopy subsystem, calculates the increment to canopy water storage based on current storages and rainfall. Water passes through the canopy by not being intercepted or by dripping. The two canopy water storage components are subject to evaporation at varying rates. Modified evaporative demand and total thrufall are couplings to other subsystems.

FIGURE 8 Identification of variables under decomposition for the hydrology model
a) Decomposition of S_{11} into S_{111} and S_{112}.

Description	Variables S_{110}	S_{111}	S_{112}
Precipitation	z_1	z_1	
Temperature	z_2	z_2	
Radiation	z_3	z_3	
Stream flow	x_1		x_1
Evaporation	x_2	$x_{13}+x_{15}$	
Transpiration	x_3		x_2
Canopy storage	x_4	x_4	
Snow storage	x_5	x_6	
Litter layer storage	x_6	x_7	
Channelized root zone storage	x_7		x_3
Non-channelized root zone storage	x_8		x_4
Subsoil storage	x_9		x_5
D and A variables	$(x_{10}-x_{15})$	(x_{16})	$(x_{11}-x_{14})$
Soil infiltration	x_{16}	$x_3=y_3$	z_2
Residual evaporative demand	x_{17}	$x_2=y_2$	z_1



Coupling Variables

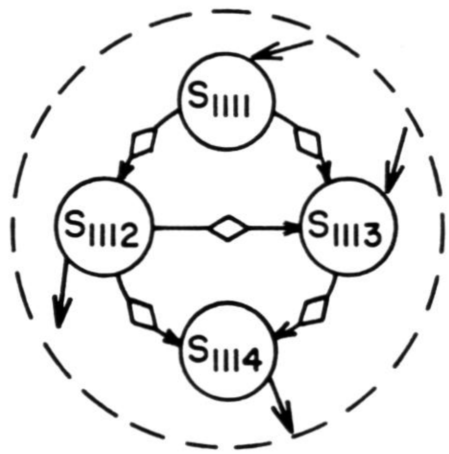
$C_{1,2}=\{(xs_{16}, y_3, z_2), (xs_{17}, y_2, z_1)\}\dagger$

Processing sequence: Ascending numerical order.

†The variable designated *xs* is the current (updated) value of *x*. This designation allows an output of one subsystem to be available in the same time step as an input to another subsystem in representing sequential relations. This *requires* that the subsystem providing the outputs be processed before that subsystem receiving the inputs.

FIGURE 8(b) Decomposition of S_{111} into subsystems.

			Variables		
Description	S_{1110}	S_{1111}	S_{1112}	S_{1113}	S_{1114}
Precipitation	z_1	z_1			
Temperature	z_2	z_2		z_1	
Radiation	z_3			z_2	
Residual evaporative demand	x_2				y_1
Soil infiltration	x_3				y_2
Canopy storage	x_4		$x_4+x_5=y_4$		
Snow storage	x_6			$x_3+x_4=y_1$	z_1
Litter layer storage	x_7				x_4
Rainfall	x_8	y_1	z_1		
Snowfall	x_9	y_2		z_3	
Evaporative demand	x_{10}	y_3	z_2		
ET demand after canopy evaporation	x_{11}		y_1		z_2
Thrufall plus drip	x_{12}		y_2	z_4	
Evaporation from canopy	x_{13}		y_3		
Litter infiltration	x_{14}			y_2	z_3
Evaporation from litter layer	x_{15}				y_3
D and *A* variables	(x_{16})	(y_4-y_7)	(y_5, y_6)		



$C_{1,2}=\{(xs_8, y_1, z_1), (xs_{10}, y_3, z_2)\}$

$C_{1,3}=\{(xs_9, y_2, z_3)\}$

$C_{2,3}=\{(xs_{12}, y_2, z_4)\}$

$C_{2,4}=\{(xs_{11}, y_1, z_2)\}$

$C_{3,4}=\{(xs_6, y_1, z_1), (xs_{14}, y_2, z_3)\}$

Processing sequence: Ascending numerical order.

S_{1113}, the snowpack subsystem, adds snowfall to the frozen part of its current snowpack. Environmental conditions determine how much will melt into free water. The amount of free water which the snowpack can retain is adjusted according to how much frozen water remains. All water which cannot be retained will infiltrate the litter. If no snowpack exists, then all thrufall infiltrates the litter.

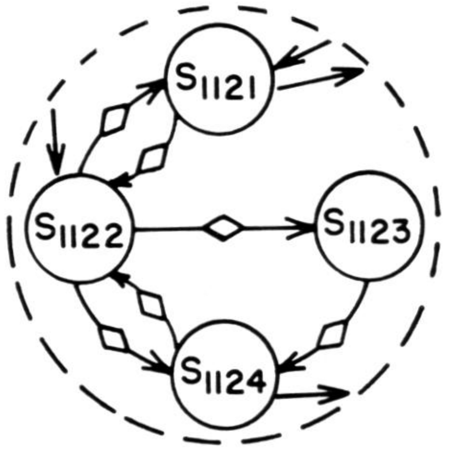
S_{1114}, the litter layer subsystem, increments its own storage depending on current storage and litter infiltration. Excess water infiltrates the soil. Duff water is then subjected to evaporation but total evaporation from the litter layer is limited so that only a fraction of the residual demand may be satisfied from here. Thus, evaporation can modify total atmospheric water demand but is unlikely to satisfy it.

In the belowground subsystem, S_{1121}, the transpiration subsystem, satisfies all remaining evaporative demand if there is sufficient water in the non-channelized root zone. The amount of transpiration which has taken place is a coupling to the non-channelized root zone subsystem.

S_{1122}, the non-channelized root zone subsystem, decides how much soil infiltration adds to

FIGURE 8(c) Decomposition of S_{112} into subsystems.

Description	S_{1120}	S_{1121}	S_{1122}	S_{1123}	S_{1124}
Residual evaporative demand	z_1	z_1			
Soil infiltration	z_2		z_1		
Stream flow	x_1				y_1
Transpiration	x_2	y_1	z_2		
Channelized root zone storage	x_3		z_3		$x_2=y_2$
Non-channelized root zone storage	x_4	z_2	$x_1=y_1$		
Subsoil storage	x_5			$x_4+x_5=y_4$	
Net input to channelized root zone storage	x_6		y_2		z_1
Percolation to subsoil	x_7		y_3	z_1	
Percolation to channelized subsoil	x_8			y_1	z_2
Subsoil contribution to stream flow	x_9			y_2	z_3
Non-accepted percolation to subsoil from root zone	x_{10}			y_3	z_4
A and *D* variables	$(x_{11}-x_{14})$			(x_4-x_6)	



$C_{1,2}=\{(xs_2, y_1, z_2)\}$ $C_{2,1}=\{(x_4, y_1, z_2)\}$

$C_{2,3}=\{(xs_7, y_3, z_1)\}$

$C_{2,4}=\{(xs_6, y_2, z_1)\}$ $C_{4,2}=\{(x_3, y_2, z_3)\}$

$C_{3,4}=\{(xs_8, y_1, z_2), (xs_9, y_2, z_3), (xs_{10}, y_3, z_4)\}$

Processing sequence: Ascending numerical order.

non-channelized root zone storage. Water either flows or is forced out of the non-channelized component into the channelized component. Total percolation is based on the current status of the channelized root zone, communicated by a coupling with that subsystem, as well as water attempting to enter. The actual storage of channelized root zone water is maintained in the channelized root zone subsystem.

S_{1123}, the subsoil system, splits percolation so that part enters the non-channelized subsoil according to its unfilled capacity and the rest flows into the channelized part. Lateral flow from the non-channelized compartment to the channelized compartment is calculated. Outflow from the channelized compartment determines the subsoil contribution to stream flow and is subject to a time lag. Knowledge of the amount entering the channelized portion from the root zone is needed for calculation of non-accepted percolation in the channelized root zone subsystem.

The belowground sequential structure was conceptualized so that the same structure could be used in a spatially stratified model. In such a model it is possible for uphill subsoil sources to supply water in excess of the capacity of the downhill strata to pass it through or store it. An excess input would result in a backflow from subsoil to root zone. This reverse flow is termed non-accepted percolation and is passed, along with percolation into the channelized subsoil, to the channelized root zone subsystem for updating of the channelized root zone storage.

S_{1124}, the channelized root zone subsystem, calculates the actual inflows and outflows of the channelized root zone compartment and updates it accordingly. In addition, all contributions to stream flow are summed and output.

This describes the finer structure of the hierarchical model. Much of the conceptual overview of the model has been treated elsewhere.[8]

Decomposition and Sensitivity Analysis

The effect of explicit decomposition on sensitivity analysis is very much dependent on the nature of the decomposition, as well as on the nature of the behavior of interest. The first, and simplest, form of decomposition follows the ordinary mathematical form: if $y=f(z)$, then $[y=f_1(c),\ c=f_2(z)]$ is a decomposition of f. In the system context, we can consider this a series decomposition, as illustrated below.

$$z \to S \to y \qquad z \to S_1 \to c \to S_2 \to y$$

Now, one obviously can address the sensitivity of the coupling variable, c, with respect to (wrt) the parameters of S_1 and the sensitivity of y wrt the parameters of S_2. It follows that appraisal of sensitivity of y to parameters of S_1 can be affected by driving S_2 by the output trajectories $\{C_k\}$ of the sensitivity runs of S_1; it is not necessary for the two systems to be coupled together in order to make these analyses. However, there will be little saving of computing time. The saving is reflected only in the fact that S_1 computations are not made for those runs designed to appraise sensitivity with respect to parameters of S_2.

It should be noted that reduction in computation time is greater when one treats pairs of parameters than when parameters are perturbed individually, and that three modules in series afford greater reduction than two. It should also be noted that it is a good strategy to analyze the sensitivity of y with respect to b_1 by runs on S, and the sensitivity of y wrt b_2 by runs on S_2. Further, the gain in conceptual understanding that follows study of the internal couplings is one of the advantages of decomposition, as is the economy of structural evolution and tuning. The latter really requires the existence of an objective trajectory for the internal coupling, c.

Parallel decomposition involves the identification of several outputs, such that a separate pathway of the system generates each, as in the following figure:

$$z \to S \to y_1,\ y_2 \qquad z \to S_1 \to y_1,\quad z \to S_2 \to y_2$$

This decomposition identifies separate and independent subsystems, each of which can be evaluated in isolation from the rest. It is in this context that we say that individual "behaviors" have a tendency to isolate on individual subsystems. Gain in computing is easily calculated for an explicit case, and derives from the elimination of the necessity to consider interaction among the parameters of different subsystems, or the possibility that any parameter will affect an output of another subsystem. A general condition is that the total number of parameters in the decomposed form may be greater than the number in the entire form, which somewhat hinders the general evaluation.

Now the sensitivity of y_1 to b_1 is evaluated by runs of S_1, the sensitivity of y_2 to b_2 by runs of S_2, and so on. If S_1 and S_2 each contain n parameters, and S contains $2n$, then the gain under decomposition for single parameter perturbations is the relative cost of running each subsystem n times and of running the whole model in $2n$ times. For parameters varied in pairs, the gain is greater, but the amount of gain depends on the nature of the perturbation analyses and on the size of the system. This can be put into intuitive perspective by the simple device, the macro-parameter, used by modellers from the Grasslands Biome.[13] If one selects one parameter from each subsystem to form a *set* identified as a macroparameter, and perturbs each element of that macroparameter simultaneously, then the results of sensitivity analysis on S of the vector of outputs with respect to the macroparameters are unambiguous under the parallel model structure. To illustrate, let $B_1 = \{b_{11}, b_{12}, \ldots, b_m\}$, $B_2 = \{b_{21}, b_{22}, \ldots, b_{2m}\}$, etc. Then sensitivity of y_1 wrt B_1 over S is interpreted as sensitivity of y_1 wrt b_{11}, and sensitivity of y_2 wrt B_1 as sensitivity of y_2 wrt b_{12}. Interaction of B_1 and B_2 wrt y_1 is interpreted as interaction of b_{11} and b_{21}, wrt y_1. In a simple $\pm\Delta$ sensitivity context, one can study all the interactions of a model of m subsystems with n parameters each by making 3^n runs of S. If the decomposition pattern is not recognized, 3^{mn} runs are required.

For two subsystems of 3 parameters each, this predicts a reduction from 729 runs to 27 runs.

Here we emphasize that the submodels do not have to be explicitly implemented and run in isolation for these gains to be achieved. One has only to recognize the decomposition and take it into account in identification of the macroparameters. Further, some of the decomposition conditions are testable from the output of such an analysis, so that it is a useful way to examine hypotheses regarding decomposition.

However, we would argue that additional advantages accrue from explicit decomposition and the implementation of the separate modules. Ease of conceptualization, and facilitation of the subjective process of changing module structure to achieve the desired behavior, are two readily apparent examples. Again, we find that existence of objective data sets for the relevant behaviors (outputs) is indispensable to the kind of activity that we want to generate in association with sensitivity analysis.

The nature of the different behaviors which may identify a parallel decomposition is also of interest. The case of uniquely identified variables is apparent; given two such variables, the decomposition process is one of constructing a parallel representation of the two behaviors. Though conceptually simple, this process may not be simple in practice; it may be necessary to invent artificial variables in order to affect the decomposition. This is illustrated by a linear model; the modal, or cannonical, variables are the artificial variables which provide the parallel, decomposition of the system. If a linear model is not fully diagonalizable, then some of the modal variables are inextricably coupled, and must be treated in the same 'subsystem."

We may infer from this that in non-linear systems one should consider the replacement of natural variables by artificial variables, and that one cannot be guaranteed that each ultimate variable will be isolated by the process. Some behaviors may be inherently multivariate, or perhaps the decomposition of some behavior must achieve a lower level of organization.

Another form of parallel decomposition is illustrated by the "spectral" decomposition of a single variable into several, representing different frequencies. Recalling Simon's position that the various hierarchical levels are characterized by response frequency,[2] we can view this form of decomposition as a means of identifying level.

$$z \to S \to y$$

$$\begin{array}{ll} z \to z_1 \to S_1 \to y_1 & \\ \searrow & y = y_1 + y_2 \\ \quad z_2 \to S_2 \to y_2 & \end{array}$$

Sensitivity Analysis of the Hydrology Model

The decomposed model was utilized for a sensitivity analysis. The objective of decomposition was to produce subsystems of considerably less complexity than the original single level model. Comparison of Figures 5, 6, and 7 reveals the manner in which complexity per system has been reduced. The FLEX version of the model actually contained the g-function detail of the subsystems; Figure 5 has been simplified for comprehensibility, and Figures 6 and 7 indicate the added detail in identification of internal variables and processes. The reduction in model dimensionality allows a fuller examination of its behavioral space at less cost.[6,12]

In making such a gain in information per run, we give up some information. By decomposition, we have *assumed* that some interactions are unimportant, and the study of these has been eliminated from the examination of behavior. But this is an expression of the observation that certain behaviors are more or less completely isolated on the subsystems, and to the degree that this is true, the information lost is minimal. To express this another way, we can note that decomposability implies that changes in some parameter values will result in virtually no change in some observed behaviors and so the behaviors need not be tested with these parameters in the sensitivity analysis.

The results of our sensitivity analyses are given as Figure 9. A subjectively chosen "best" run was used as the standard of comparison and is designated the nominal run. Driving data for each uncoupled subsystem was derived from the nominal run by outputting the essential internal variables. The diagrams of Figure 9 show the standard behavior (chosen for each subsystem) as the middle horizontal line, and sensitivity response is plotted as a percent change in this standard.

Each parameter in a subsystem is independently increased and decreased by a subjectively chosen percentage of its value in the nominal run. Changes in subsystem behaviors are plotted

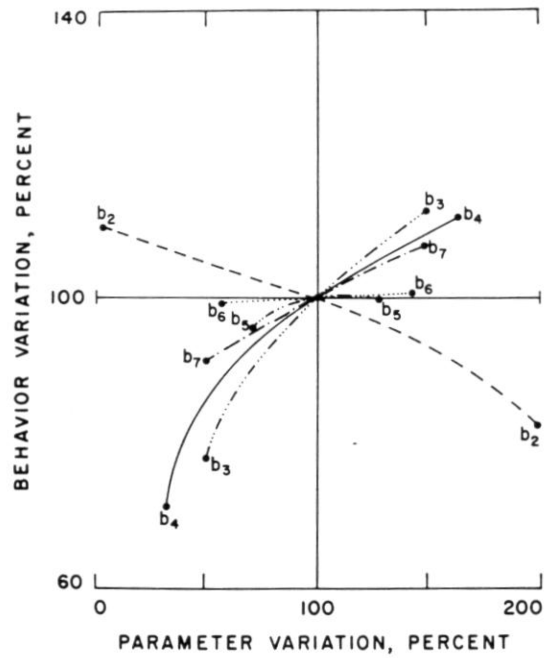
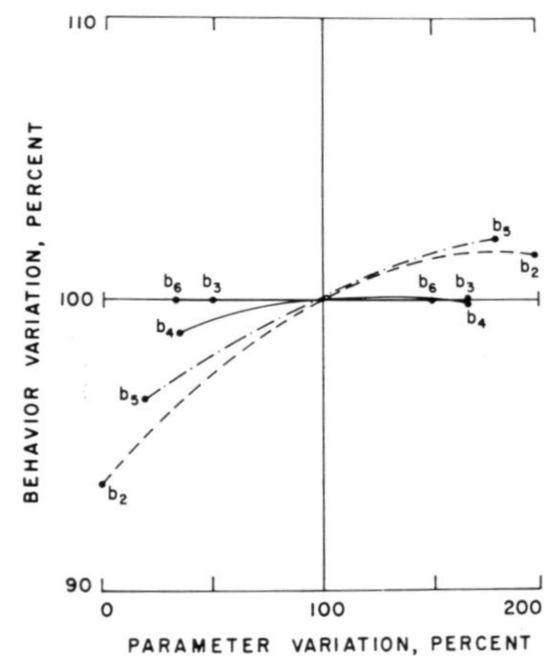


FIGURE 9 Results of the sensitivity analysis. Each subsystem is run with each parameter changed independently by a subjectively chosen percentage of its nominal value. All runs are Watershed 2 runs using driving data for the year July 1, 1959 to June 30, 1960.

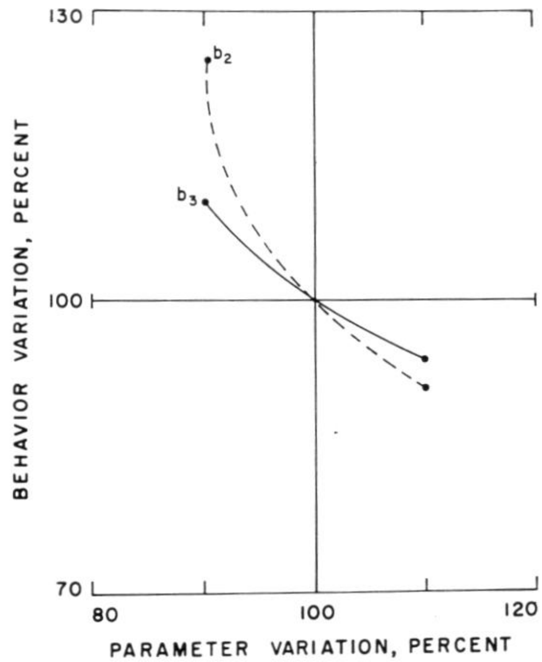
a) Sensitivity of parameters in the canopy subsystem. The standard for behavioral comparison is 26.649 cm of annual evaporation.

Parameter	*Description*
b_2	Proportion of rain direct to forest floor.
b_3	Maximum total canopy storage.
b_4	Proportion of canopy storage on foliage
b_5	Proportion of interception by foliage.
b_6	Rate of evaporation from foliage.
b_7	Proportion of remaining ET demand satisfied by epiphytes.



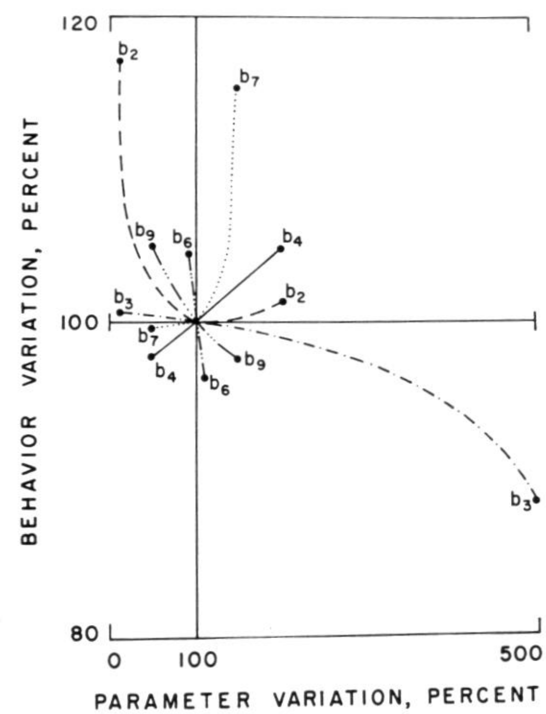
b) Sensitivity of parameters in the litter layer subsystem. The standard of comparison is 28.501 cm of annual evaporation. Note: The behavioral value at $b_2=0$ is the standard for the canopy subsystem.

Parameter	*Description*
b_2	Maximum storage in litter layer.
b_3	Evaporation rate from litter layer.
b_4	Maximum proportion of litter layer water which can evaporate in one day.
b_5	Maximum proportion of remaining ET demand which can be satisfied by evaporation of litter layer water.
b_6	Rate at which litter layer infiltration increments storage.



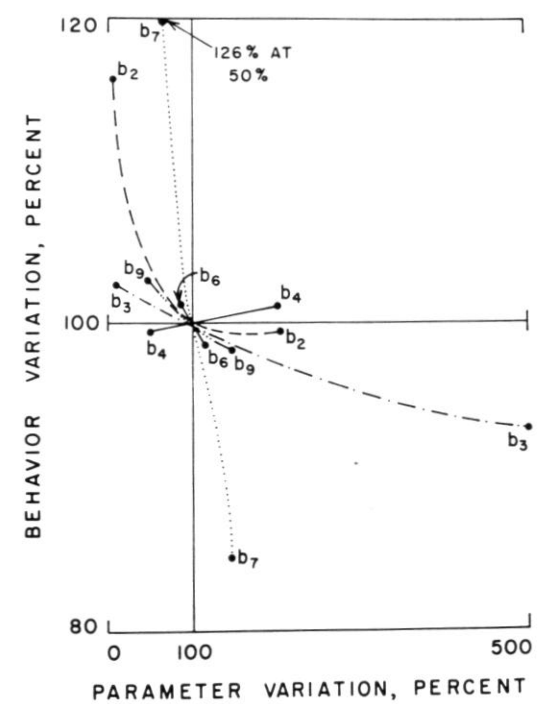
c) Sensitivity of parameters in the transpiration subsystem. The standard of comparison is 50.279 cm of annual transpiration.

Parameter	*Description*
b_2	Wilting point.
b_3	Point of transpiration resistance.

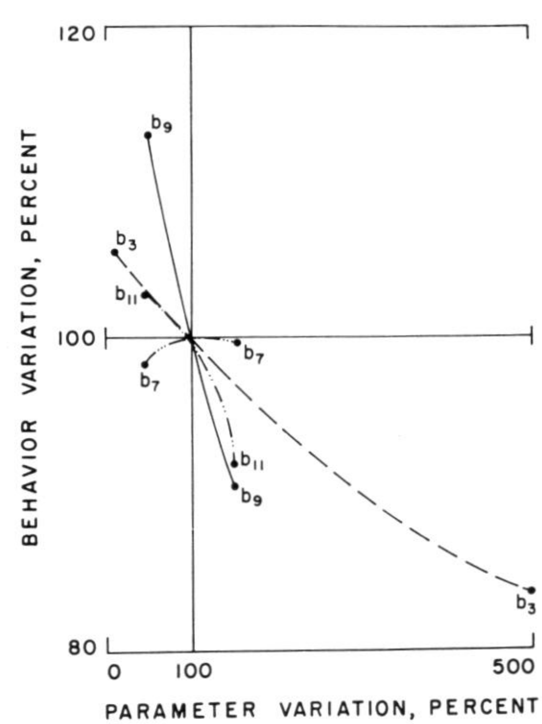


d) Sensitivity of the parameters in the root zone subsystems. Standard of comparison is $\theta_1 = \Sigma(y-\hat{y})^2 = 9.450 \times 10^5$ for two years (July 1, 1958 to June 30, 1960) where y is observed stream flow and $\hat{y}$ is estimated stream flow.

Parameters in S_{1122}	*Description*	*Correspondence in* S_{1124}
b_2	Channelized root zone water daily flow rate.	b_2
b_3	Non-channelized root zone water daily flow rate.	None
b_4	Proportion of root zone which is channelized.	b_4
b_6	Root zone storage capacity.	b_6
b_7	Proportion of percolation into non-channelized root zone storage.	None
b_9	Resident time factor for infiltration into channelized root zone.	None

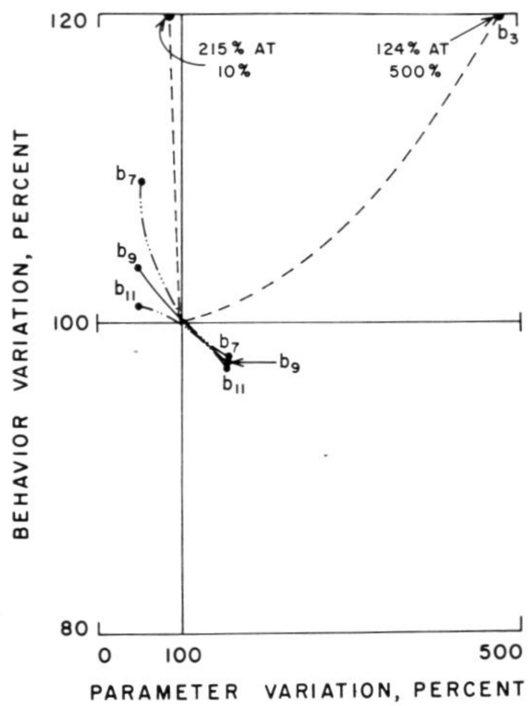


e) Sensitivity of parameters in the root zone subsystems. Standard of comparison is $\theta_2 = \Sigma(\ln y/\hat{y})^2 = 3.985 \times 10^2$ for two years (July 1, 1958 to June 30, 1960) where y and $\hat{y}$ are as defined in 9(d). Parameters are identical with those in 9(d).



f) Sensitivity of parameters in the subsoil subsystem. Standard of comparison is θ_1 as defined in 9(d).

Parameters	*Description*
b_3	Non-channelized subsoil daily flow rate.
b_7	Proportion of percolation which is into non-channelized subsoil.
b_9	Resident time factor for percolation into channelized subsoil.
b_{11}	Spatial weighing factor used in lag function.



g) Sensitivity of parameters in the subsoil subsystem. Standards of comparison is θ_2 as defined in 9(e) and parameters are identical with those in 9(f).

versus the percentage change in parameter value and a curve is drawn to connect the two points through the common point of the nominal run in the center of the diagram. Each curve represents a two-dimensional plane passed through the multi-dimensional response hypersurface. Each response profile is then rotated and all are projected onto a common two-dimensional surface for comparison. Because only three points are used, the profile is an indication of the response and not an exact rendition.

Conclusion

Current theoretical perspectives that complex systems are hierarchically organized stimulate the desire to model complex systems hierarchically. The systems theory of G. Klir[9] provides a general theoretical basis for such models, but not the necessary details for model implementation. Specific details are provided by the FLEX paradigm.[6,8,12] In this paper, we have presented some details of the practice of modelling using the FLEX convention and processor,[8] with certain considerations of the advantages accruing to a hierarchical approach. Four conceptual strategies of hierarchical modelling are identified, and discussed in the context of development of a hydrology model.

It is a happy result that the most productive modelling strategy integrates the analytic and synthetic approaches. It demonstrates that the argument between the reductionist/mechanistic school and the vitalist/holistic school is based on the orientation toward hierarchies of the members of each school. Those involved in looking outward see emerging behaviors unpredictable if the system is examined in less than its totality. Those involved in looking inward see all behaviors as ultimately explainable in terms of the properties of the parts and their interactions.

Both views are correct and both are necessary. Taken together, they allow one to build a hierarchical model which may be viewed as a strict tree. In this manner, we are freed from the constraint of always viewing the informal hierarchy in one direction, either inward or outward, but may search in both directions as appropriate. The flexibility which this provides is absolutely necessary if we are to strive toward creative solution processes.

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For biographies and photographs of the authors, please see Vol. 4, No. 2, p. 137, of this journal.

APPENDIX

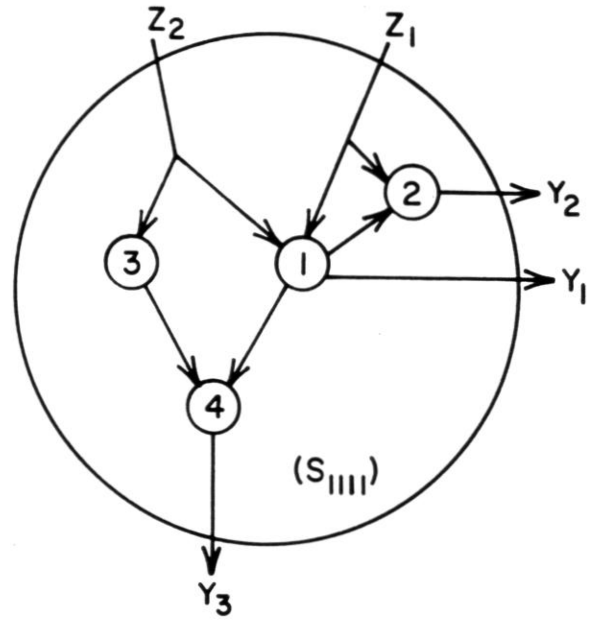
In the appendix is included a summary of each of the eight lower level subsystems of the hierarchical hydrology model. Figure 8 is especially useful in cross referencing the FLEXFORM summaries, and Figures 6 and 7 are useful in conceptualizing the manner of coupling.

The complete FLEXFORM of this model is on deposit at the General Systems Depository, and available to the interested reader from that source.

APPENDIX

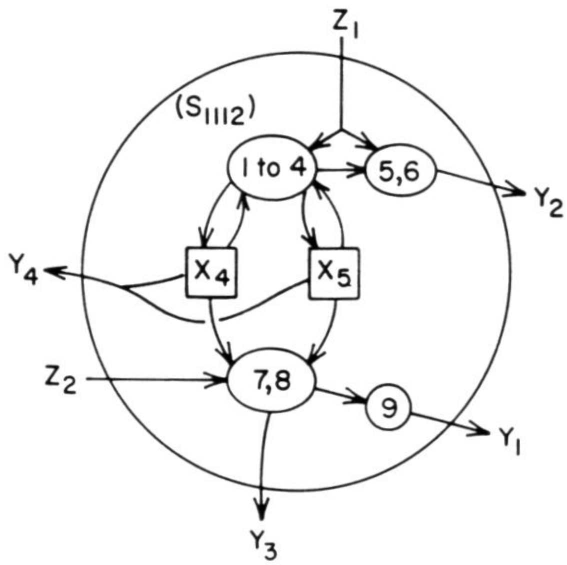
Summary Flexform
Model: S_{1111}

Title: *Meteorological subsystem*
Date: *December 23, 1976*



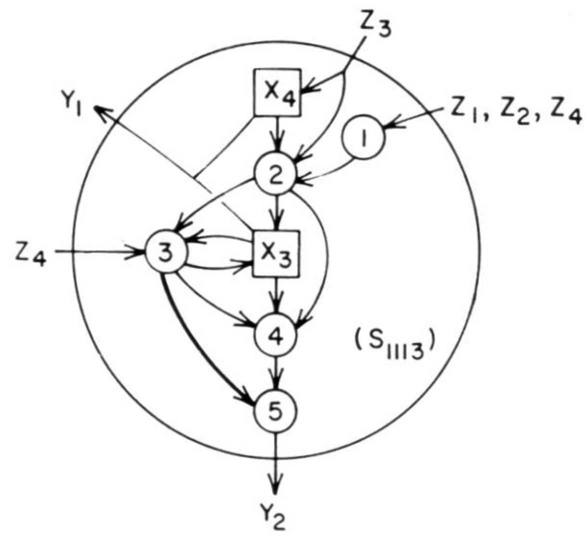
Variables	*Description*	*Ghost Coupling*
z_1	Daily total precipitation	z_1
z_2	Average daily air temperature	z_2
y_1	Rainfall	x_8
y_2	Snowfall	x_9
y_3	Evaporative demand	x_{10}
g_1	Precipitation as rain	
g_2	Precipitation as snow	
g_3	Potential evapotranspiration	
g_4	Adjusted potential evapotranspiration.	

APPENDIX
Summary Flexform
Model: S_{1112}
Title: *Canopy subsystem*
Date: *December 23, 1976*



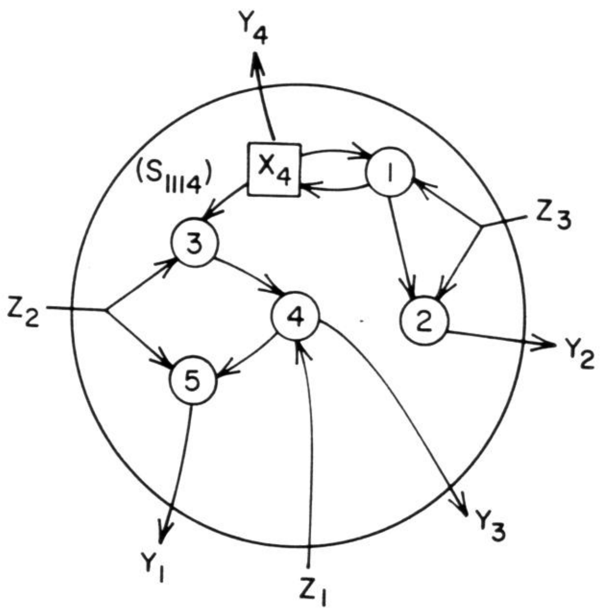
Variables	*Description*	*Ghost Coupling*
z_1	Rainfall	xs_8
z_2	Evaporative demand	xs_{10}
y_1	ET demand after canopy evaporation	x_{11}
y_2	Thrufall plus canopy drip	x_{12}
y_3	Canopy evaporation	x_{13}
y_4	Canopy storage	x_4
x_4	Foliage storage	
x_5	Epiphyte storage	
g_1	Foliage charge rate	
g_2	Epiphyte charge rate	
g_3	Incremental input to foliage	
g_4	Incremental input to epiphytes	
g_5	Canopy drip	
g_6	Thrufall	
g_7	Evaporation from foliage	
g_8	Evaporation from epiphytes	
g_9	ET demand after canopy evaporation	

APPENDIX
Summary Flexform
Model: S_{1113}
Title: *Snowpack subsystem*
Date: *December 23, 1976*



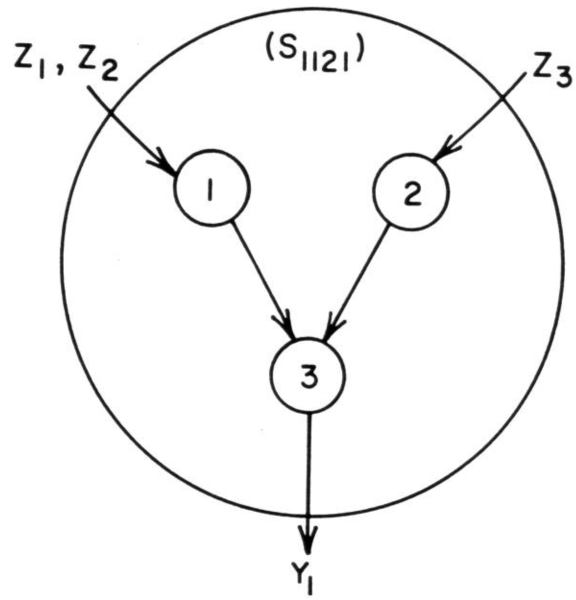
Variables	*Description*	*Ghost Coupling*
z_1	Average daily air temperature	z_2
z_2	Radiation	z_3
z_3	Snowfall	xs_9
z_4	Thrufall plus drip	xs_{12}
y_1	Snow storage	x_6
y_2	Litter infiltration	x_{14}
x_3	Free water snow storage	
x_4	Frozen water snow storage	
g_1	Potential snow melt	
g_2	Actual snow melt	
g_3	Thrufall incremental input to free water storage	
g_4	Free water leaving snow pack	
g_5	Litter infiltration	

APPENDIX
Summary Flexform
Model: S_{1114}
Title: *Litter layer subsystem*
Date: *December 23, 1976*



Variables	*Description*	*Ghost Coupling*
z_1	Snow storage	xs_6
z_2	ET demand after canopy evaporation	xs_{11}
z_3	Litter infiltration	xs_{14}
y_1	Residual evaporative demand	x_2
y_2	Soil infiltration	x_3
y_3	Evaporation from litter layer	x_{15}
$x_4 = y_4$	Litter layer storage	x_7
g_1	Litter layer incremental input	
g_2	Soil infiltration	
g_3	Potential evaporation	
g_4	Actual evaporation	
g_5	Residual evaporative demand	

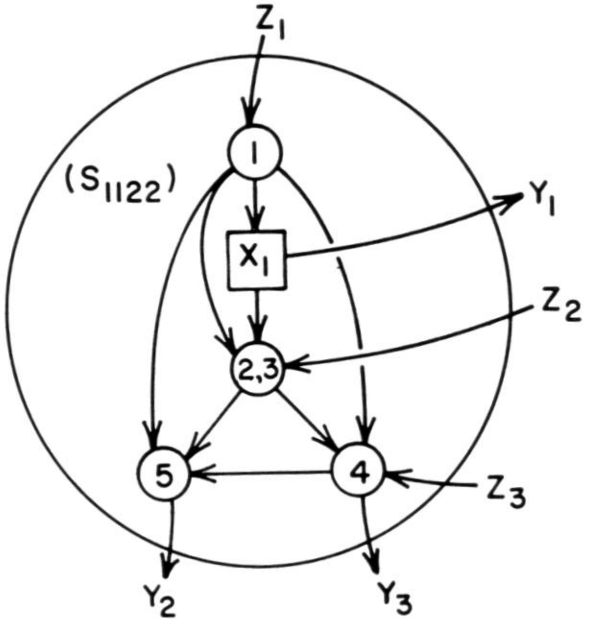
APPENDIX
Summary Flexform
Model: S_{1121}
Title: *Transpiration subsystem*
Date: *December 23, 1976*



Variables	*Description*	*Ghost Coupling*
z_1	Residual evaporative demand	z_1
z_2	Non-channellized root zone storage	x_4
z_3	Seasonal transpiration capacity	
y_1	Transpiration	x_2
g_1	Potential transpiration	
g_2	Seasonal transpiration capacity	
g_3	Actual transpiration	

APPENDIX
Summary Flexform
Model: S_{1122}

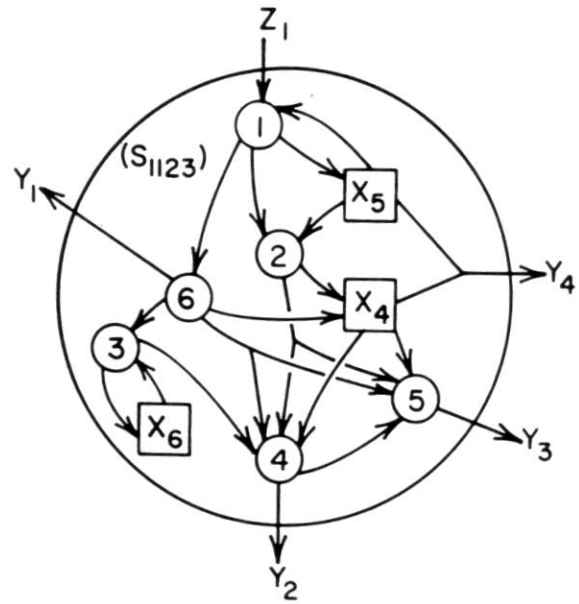
Title: *Non-channelized root zone subsystem*
Date: *December 23, 1976*



Variables	*Description*	*Ghost Coupling*
z_1	Soil infiltration	z_2
z_2	Transpiration	xs_2
z_3	Channelized root zone storage	x_3
$x_1 = y_1$	Non-channelized root zone storage	x_4
y_2	Net input to channelized root zone storage	x_6
y_3	Percolation to subsoil	x_7
g_1	Infiltration into non-channelized root zone	
g_2	Non-channelized root zone water lateral flow	
g_3	Overflow into chanelized root zone storage	
g_4	Percolation from channelized root zone	
g_5	Net input to channelized root zone storage	

APPENDIX
Summary Flexform
Model: S_{1123}

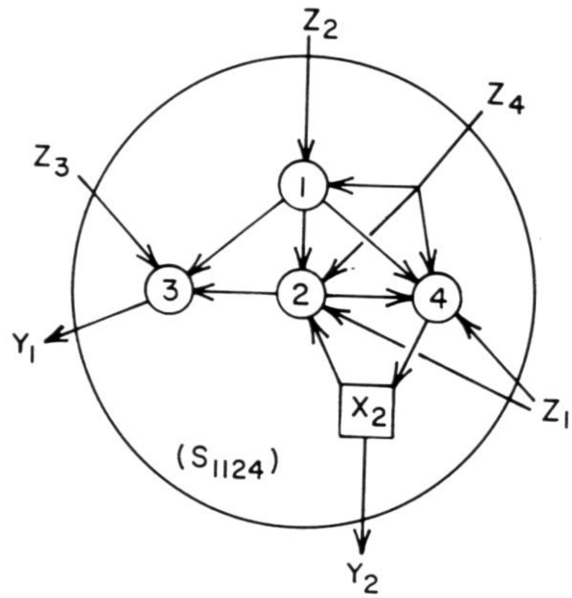
Title: *Subsoil subsystem*
Date: *December 23, 1976*



Variables	*Description*	*Ghost Coupling*
z_1	Percolation to subsoil	xs_7
y_1	Percolation to channelized subsoil	x_8
y_2	Subsoil contribution to stream flow	x_9
y_3	Non-accepted percolation	x_{10}
y_4	Subsoil storage	x_5
x_4	Channelized subsoil storage	
x_5	Non-channelized subsoil storage	
x_6	Lagged channelized subsoil water	
g_1	Percolation to non-channelized subsoil	
g_2	Non-channelized subsoil water lateral flow	
g_3	Current lag effect	
g_4	Channelized subsoil water lateral flow	
g_5	Non-accepted percolation to subsoil from root zone	
g_6	Percolation to channelized subsoil	

APPENDIX
Summary Flexform
Model: S_{1124}

Title: *Channelized root zone subsystem*
Date: *December 23, 1976*



Variables	*Description*	*Ghost Coupling*
z_1	Net input to channelized root zone storage	xs_6
z_2	Percolation to channelized subsoil storage	xs_8
z_3	Subsoil contribution to stream flow	xs_9
z_4	Non-accepted percolation to subsoil from root zone	xs_{10}
y_1	Stream flow	x_1
$x_2 = y_2$	Channelized root zone storage	x_3
g_1	Channelized root zone water lateral flow	
g_2	Channelized root zone water overflow	
g_3	Estimated stream flow	
g_4	Updated net input to channelized root zone storage	