Water Balance Validation of a Temperature-Dependent Parameter Value of the Priestley-Taylor Equation of Evapotranspiration

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Abstract: New research suggests that the Priestley-Taylor (PT) parameter, α, displays a temperature dependency with a significant increase in its generally accepted constant value of 1.26 as air temperature (T) drops. Multi-year water-balance data from two humid, cool climate experimental watersheds –Hubbard Brook in New Hampshire and Lookout Creek in Oregon, USA— and from a warm, humid catchment at the border of Mississippi and Louisiana as a control, support a temperature-dependent α(T) value. Mean annual evapotranspiration rates obtained with α(T) were superior to those of α=1.26 and fell within 3% of the water-balance derived (precipitation minus runoff) values. As a consequence, humid, cold-region evaporation/evapotranspiration may be significantly underestimated via the PT equation when the classical value of 1.26 is employed in place of a temperature-modulated α(T).

Keywords: *Evapotranspiration, evaporation, Priestley-Taylor equation, Priestley-Taylor parameter.*

1. Introduction

The Priestley-Taylor (PT) equation (1972) describes the evapotranspiration (ET) rate of a wet environment when water availability at the surface is not limiting

$$\mathrm{ET} = \alpha \frac{\Delta}{\Delta + \gamma} \mathrm{R_n} \tag{1}$$

Here R_n is the available energy (i.e., net radiation) at the wet surface, specified in water depth per unit time (e.g., mm d^{-1}), Δ is the slope of the saturation vapor pressure curve at the air temperature (T), and γ [= $c_p p$ / (0.622L)] is the psychrometric constant, where c_p is the specific heat of air at constant pressure (p) and L is latent heat of vaporization for water. The coefficient α is generally accepted to express the evaporation-enhancing effect of large-scale entrainment of drier free-tropospheric air resulting from the growing daytime convective boundary layer (Brutsaert, 1982; deBruin, 1983; Culf, 1994; Lhomme, 1997; Heerwaarden et al., 2009). Employing warm season (11 °C < T < 33 °C) data from both hemispheres, Priestley and Taylor (1972) found that α assumes a value of about 1.26, which has been used ever since in the literature as the standard value of the PT parameter.

Recently Szilagyi et al. (2014), employing ERA-Interim global reanalysis data (http://www.ecmwf.int/products/), found that the value of α changes with temperature (T), and specified the best-fit polynomial (Figure 1) as presented in equation 2.

$$\alpha(\mathrm{T}) = -3.89 \cdot 10^{-6}\mathrm{T}^3 + 4.78 \cdot 10^{-4}\mathrm{T}^2 - 2.54 \cdot 10^{-2}\mathrm{T} + 1.64 \tag{2}$$

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Equation 2 is valid in the 0 – 30 °C range and yields an α value of 1.2 at 30 °C, and 1.64 at 0 °C, which is a significant difference.

The above equation has not been validated in cold/cool climates by comparing PT-derived ET rates using the classical α = 1.26 value or the current $\alpha(T)$ relationship. Therefore water balance data of two humid, cool-climate experimental watersheds plus those of a warm, humid catchment (as control) are to be employed and the multi-year water-balance derived ET rate as the difference in precipitation (P) and runoff (Q) compared to the PT-predicted mean annual ET fluxes.

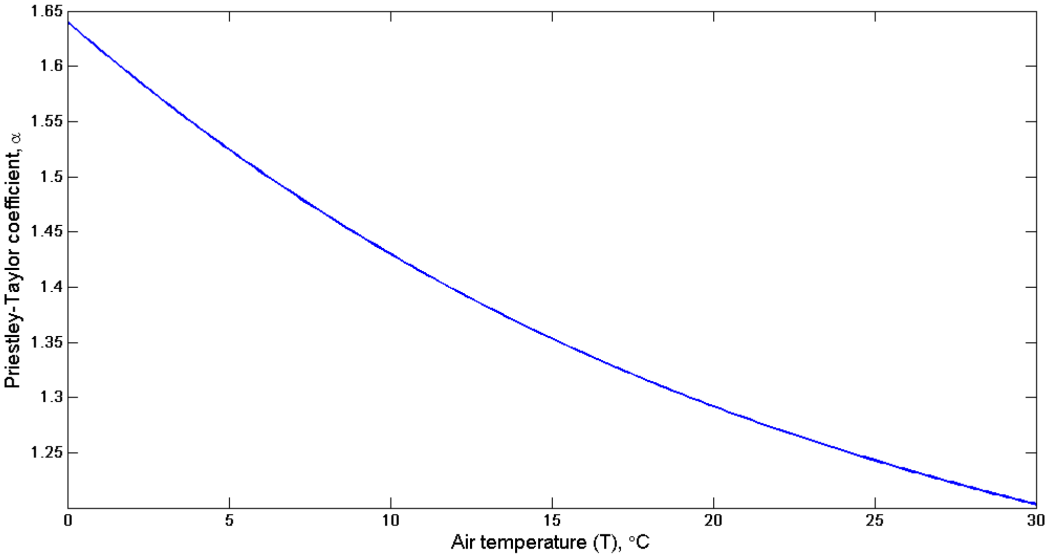

Figure 1 Graph of equation 2 over its temperature range of applicability (Szilagyi et al., 2014)

2. Description of the watersheds

Out of the many experimental watersheds with detailed hydrological and multi-site meteorological data, maintained either by the Agricultural Research Service of the US Department of Agriculture (ars.usd.gov) or the Long Term Ecological Research Network (www.lternet.edu), only two were found (i.e., Hubbard Brook and Lookout Creek) situated in a humid, cool climate that had the necessary multi-year data of air and dew-point temperature, solar radiation, precipitation and stream discharge (Figure 2).

The Hubbard Brook experimental watershed (31.6 km^2) within the White Mountain National Forest in New Hampshire is made up of several forested sub-catchments for which multi-year, accurate daily meteorological measurements are available, however runoff is not measured from the main watershed itself. A common time-period for which all required data were available for altogether eight sub-catchments (at altitudes between 442 and 905 m) was found to be May 1, 1985 till April 30, 2009. The eight sub-catchments are predominantly exposed either to the south or north. While precipitation has been measured at several locations adjacent to the sub-catchments, solar radiation is measured only at one central location resulting in only one multi-year mean annual PT ET estimate. The watershed enjoys about 1400 mm precipitation a year, evenly distributed among the months. The winter periods (when freezing may occur) are long and cold. January averages are about -9 °C, but long periods of -12 °C to -18 °C are common. Occasional midwinter thaws can result in elevated streamflow. The average July temperature is ~18 °C during the short and cool summers.

Lookout Creek (drainage area of 64 km^2) is found in the Andrews Experimental Forest of the western Cascade Range of Oregon at elevations from 410 to 1630 m. Streamflow is measured at several sub-catchments, most of them occupying only a usually steep mountain slope, beside the main outlet. Both precipitation and other meteorological variables (including solar radiation) are measured at several locations (six for temperatures and four for solar radiation) within the basin, but typically at valley locations which makes obtaining accurate radiation balances for the steep slopes difficult. Within the Lookout Creek watershed one larger sub-catchment with monitored flow exists, called Mack Creek with a drainage area of 5.81 km^2.

The maritime climate has wet, mild winters and drier, cool summers. At the main meteorological station at 430 m elevation, mean monthly temperatures range from 1°C in January to 18 °C in July. Precipitation falls primarily from November through March, and varies with elevation from 2300 to over 3550 mm at higher elevations. In the wintertime, rain is mixed with snow in the lower portion of Lookout Basin and snow is more persistent at higher elevations (above 1000 m). The time-period with the most abundant data falls between October 1, 1998 and September 30, 2011. See www.lternet.edu for more detailed information of the two experimental catchments.

Figure 2 Location of the watersheds employed in the study. HB: Hubbard Brook in New Hampshire; LC: Lookout Creek in Oregon, and; AR: Amite River near Denham Springs in Louisiana

As a control case, a third, warm-climate humid watershed (shared by Mississippi and Louisiana), minimally affected by human alterations, the Amite River near Denham Springs, LA (drainage area of 3,315 km^2 with a mean elevation of ~100 m), was also selected, for analysis of its 30-year runoff data together with the similarly long Solar and Meteorological Surface Observation Network (SAMSON) data set from nearby Baton Rouge, LA. The watershed enjoys about 1500 mm precipitation a year, more or less evenly distributed among the months. Winter periods are short and mild with January averages of 10 °C, while July temperature is around 27 °C during the long and hot summers.

3. Estimation of mean annual evapotranspiration rates

The Hubbard Brook website specifies mean annual ET as about 500 mm, while the Andrews Forest LTER site does not disclose it. Since at Hubbard Brook streamflow is not measured at the main outlet, watershed-representative water balance ET (ET_{wb}) could only be obtained as the arithmetic mean of the eight $P - Q$ sub-catchment values (Table 1). The so-obtained 493 mm annual ET rate is very close to what the site specifies. Note that the long period of about 25 years allows for not using hydrologic years (i.e., from October till November), as performed for Lookout Creek where the time-period is about half of what is available at Hubbard Brook. Due to the somewhat milder climate and more abundant precipitation, mean annual water balance ET rates became 632 mm for Mack Creek and almost the same, 638 mm for Lookout Creek, which encompasses the former. The presence of deep valleys between steep slopes largely hinders the estimation of individual sub-catchment ET rates either with the water-balance approach or the PT equation. This is so because neither the closest valley bottom precipitation nor the similarly situated solar radiation measurements are representative of these steep, long slopes that contain the small sub-catchments. The meteorological measurements, especially when obtained from multiple locations, are more representative of larger, more complex catchments that comprise of many slopes with different length, steepness and aspect, and also significant valley segments, such as found at Mack Creek and at the larger Lookout Creek.

Application of the PT equation requires the net radiation term, R_n. The estimation method (called WREVAP) of Morton et al. (1985) was employed for this task at a monthly time-step since daily R_n estimates are typically laden with large uncertainties. The estimates become significantly improved for time periods of five days or longer (Morton et al., 1985). A recent study by McMahon et al. (2013a, b) found the WREVAP program the most reliable practical ET estimation tool for monthly or annual time-scales, therefore the software's net radiation outputs were also applied in the present study. Table 1 includes the WREVAP ET values (ET_M), as a by-product as well. In the winter months with near freezing-point temperatures the estimated R_n values often become negative. Equation 1 in such months would yield negative ET rates therefore ET was set to zero when this happened.

In the lack of pressure values, p in the γ term of equation 1 was estimated by the barometric formula as

$$p = 1013\left(\frac{T + 273.16}{T + 273.16 + \lambda z}\right)^{\frac{g}{\lambda R_a}} \quad (3)$$

where g is gravitational acceleration, R_a the specific gas constant of air, λ the dry adiabatic lapse rate (0.0065 Km^{-1}), and z the altitude. Equation 3 is not sensitive to the air temperature T, therefore it was applied with a constant T = 15 °C for all watersheds, resulting in $\gamma \approx 0.6$ for Hubbard Brook ($z \approx 700$ m), in 0.58 for Lookout Creek ($z \approx 1100$ m) and in 0.65 for the Amite River ($z \approx 100$ m).

Szilagyi and Jozsa (2008), Szilagyi et al. (2009), and Szilagyi (2014) proposed the evaluation of Δ at the wet-environment air temperature (T_w) estimated by a recursive algorithm in place of the actual air temperature of equation 1. This is not necessary for the current watersheds due to their deliberate selection for humidity (i.e., for the applicability of the PT equation), since under such conditions actual air temperature, T is close T_w.

Table 1 Mean annual ET estimates by method (ET_{wb} — water balance; $ET_{\alpha(T)}$ — equations 1 and 2; $ET_{\alpha=1.26}$ — equation 1; ET_M — WREVAP model) and location

	Hubbard Brook, NH (05-01-85 – 12-31-09)	Lookout Creek, OR (10-01-98 – 9-30-11)		Amite River near Denham Springs, LA (01-01-61 – 12-31-90)
	Mean of eight sub-catchments ($mm\ yr^{-1}$)	Mack Creek (closest two met. stations) ($mm\ yr^{-1}$)	Entire watershed (all met. stations) ($mm\ yr^{-1}$)	Met. station at Baton Rouge ($mm\ yr^{-1}$)
ET_{wb}	493	632	638	999
$ET_{\alpha(T)}$	481	606	626	1134
$ET_{\alpha=1.26}$	438	546	540	1128
ET_M	461	490	569	938

4. Discussion and conclusions

Equation 1 with the classical constant α value of 1.26 significantly underestimates both cool-climate water-balance derived mean annual ET rates by about 10% for Hubbard Brook and by 15% for Mack and Lookout Creeks. The underestimation greatly improves with the application of the temperature-dependent parameter value, to less than 3% for Hubbard Brook and Lookout Creek and to 4% for Mack Creek. These values are well within the combined accuracy of the precipitation and runoff measurements. WREVAP yielded ET estimates with accuracies intermediate of the two versions of equation 1 for Hubbard Brook and Lookout Creek, and gave the worst performance of all for Mack Creek. WREVAP, due to the application of the complementary relationship of evaporation (Bouchet, 1963) to predict actual ET rates, is sensitive to the vapor pressure deficit value. The low number (only two) of meteorological stations applied over a highly variable topography, neither within the boundary of Mack Creek, may explain this large error.

Table 1 illustrates well the importance of employing a temperature-dependent formulation of the Priestley-Taylor coefficient, α, when applied with low temperatures, since under such conditions the resulting Priestley-Taylor ET rates may differ by more than 20% (i.e. 1- 1.26 / 1.6) from those obtained by the classical constant value, leading to a substantial underestimation of the wet-environment latent heat fluxes in cold regions in the latter case.

For warm and humid climates the classical 1.26 value of α yields practically the same ET rates as the $\alpha(T)$ case (1128 vs 1134 mm yr^{-1}) due to a) the milder slope of the $\alpha(T)$ polynomial (equation 2) for warmer air temperatures (Figure 1) and, b) the fact the 1.26 value is more or less an average value of $\alpha(T)$ for temperatures between 15 and 30 °C, the predominant temperature range for the Amite River catchment. Both ET estimates are about 11% higher than the water-balance derived value of about 1000 mm yr^{-1}. Due to the high temperatures in summer and early fall, water may occasionally become a limiting factor for the very intensive ET rate specified by equation 1, reflected in the slight overestimation of the water-balance derived value. The Morton ET method, due to its complementary relationship, detects such water supply shortages via the vapor pressure deficit and yields a reduced ET rate, although somewhat smaller (by 6%) than the actual water-balance value. These latter results also indicate that the significant underestimation of ET by the PT equation when α = 1.26 for the cool-climate watersheds is not the result of an underestimated net radiation term, R_n, of equation 1, since then the same underestimation should have been expected for this warm-climate catchment.

Based on the above, a temperature-modulated version of the Priestley-Taylor coefficient, α, as specified by equation 2, is always recommended over the classical constant value of 1.26 when applying the PT equation for practical estimates of the wet-environment evapotranspiration rates, especially for cool or cold climates.

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