# THE DERIVATION AND VALIDATION OF A NEW MODEL FOR THE INTERCEPTION OF RAINFALL BY FORESTS

# WILLIAM J. MASSMAN\*

Department of Biology, University of Oregon, Eugene, OR 97403 (U.S.A.) (Received June 16, 1982; revision accepted November 17, 1982)

# ABSTRACT

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A dynamic model and an analytical model derived therefrom for predicting the gross interception of forest canopies are described. The dynamic model is validated using a running balance in time between rainfall, throughfall, evaporation and changes in the canopy storage. The canopy storage is increased by rainfall interception and depleted by evaporation and drainage. The evaporation rate varies with the amount of water in the canopy and is estimated by numerical methods because sufficient meteorological data are unavailable. Drainage rate expressions, similar to Rutter's exponential relationship between drip rate and water storage, are shown to be inadequate during the period of rainfall. A new drip expression which explicitly includes the rain rate, in addition to the stored water, is proposed and tested. This new expression fits the observed drip rate better and gives significantly better model prediction with fewer empirical parameters than the exponential form of drip rate. However, because one of the drip parameters in the dynamic model varied from one storm to the next and is a complicted function of rainfall characteristics, the dynamic model was simplified to an analytical model for predicting the gross interception loss. This analytical expression is easier to use because it is not sensitive to the exact value of the drip parameters; it predicted the total observed gross interception loss for the 20 storms tested to within 4%. It is suggested that the new drip expression used in the dynamic model describes better the dislodgement of previously intercepted rain droplets by falling rain droplets. Finally, a new model for estimating evaporation rates from forest canopies is proposed and discussed.

#### INTRODUCTION

Interception and evaporation of rainfall are important hydrologic processes in forested areas and have been the subject of two types of studies. The first type I shall term static models. For these models, results are presented as empirical equations relating gross interception loss to various characteristics of rainfall events. Gross interception loss is defined here as the depth of water intercepted by the canopy, then subsequently evaporated (Rutter, 1975). The second type I shall term dynamic models. For these models, differential equations are developed with the rate of change of water inside the forest canopy being given by the difference between interception, evaporation and throughfall.

<sup>\*</sup>Present address and address for correspondence: Applied Research Corporation, 8201 Corporation Drive, Suite 920, Landover, MD 20785, U.S.A.

There are many examples of static models. Zinke (1967) uses a linear regression equation of the form

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$$I = a_1 P_G + a_2 \tag{1}$$

where I is the gross interception loss,  $P_G$  is the gross precipitation, and  $a_1$ and  $a_2$  are regression coefficients. This empirical equation was given a sound physical basis by Gash (1979) who related the regression coefficients to average rainfall and evaporation rates, and to parameters describing the canopy structure: the canopy capacity (the amount of water on the canopy when rainfall and throughfall have ceased and the canopy is saturated), the free throughfall coefficient (that proportion of the rain which falls directly through the canopy without striking a surface), and the stemflow coefficient (that proportion of the rain which is diverted to the trunks). Merriam (1960) proposed an equation of the form

$$I = S_c [1 - \exp(-P_G / S_c)] + E_0 T$$
(2)

where  $S_c$  is the canopy capacity,  $E_0$  is the mean evaporation rate during the storm and T is the storm duration from the time the rain begins until it ceases. Kohler (1961) explored the physical basis for this model of gross interception loss and concluded that evaporation rates must be small compared to interception rates in order for it to be valid. There is also a conceptual problem with (2). Consider a storm in which evaporation during the storm can be ignored, i.e.,  $E_0 = 0$ , and the gross precipitation is just enough to wet the canopy:  $P_G = S_c/(1-p)$ , where p is the free through fall coefficient. Merriam's model would predict that  $I = S_c \{1 - \exp[-1/(1-p)]\} =$  $0.65S_c$  for p = 0.05. However, I should equal  $S_c$ , since it was assumed that the storm was just sufficient to wet the canopy and evaporation was negligible. Thus, (2) does not appear to conserve mass, a failing especially noticeable for small storms. Using observational data for eight small trees, Aston (1979) derived a new version of (2) which can be expressed by replacing  $P_G$  in (2) with  $(1-p) P_G$ . However, his work does not remove the conceptional difficulty pointed out earlier.

Czarnowski and Olszewski (1968) used an empirical expression similar to (2) which is given as follows

$$I = a_1 [1 - \exp(-a_2 P_G)]$$
(3)

where  $a_1$  and  $a_2$  are empirical constants found by optimization techniques. Bultot et al. (1972) suggested the following equation

$$I = (a_1 P_G + a_2 P_G^2) [a_3 / (a_4 + R_0)] (a_5 E_0 + a_6)$$
(4)

where  $a_1, \ldots, a_6$  are empirical constants derived by curve fitting using optimization techniques and  $R_0$  is the average rain rate during the storm. These last two models are less rigorous and more heuristic than either of the two previous models. Jackson (1975) tested several models and found that the following semilogarithmic curve fitted his data slightly better than other models

# $I = \mathbf{a}_1 + \mathbf{a}_2 \ln R_0 + \mathbf{a}_3 \ln T$

where  $a_1$ ,  $a_2$  and  $a_3$  are again empirical constants derived by optimization techniques. The physical basis for this expression may be found in the limiting forms of more complex dynamic models (Jackson, 1975; Massman, 1980).

While static models are simple and easy to use, they do not always give satisfactory quantitative results when the coefficients are empirically determined by a regression against a set of data (Jackson, 1975). Furthermore, any empirical results for a specific forest may not be valid for other forests in other areas (Gash, 1979). Dynamic models, on the other hand, minimize the hazards of empiricism by relying upon more fundamental physical reasoning. However, they usually require automated meteorological data stations with frequent rain rate and throughfall rate measurements, and frequent meteorologically based estimates of evaporation. These models are typically expressed as a water balance equation in the form of a differential equation. There are traditionally two approaches to solving this differential equation. The first approach involves integrating the mass balance equation analytically. The models of Merriam (1960), Kohler (1961) and Gash (1979) are examples of this approach. The second approach uses a computer to solve the water balance equation numerically. Examples of this second approach are Rutter's model (Rutter et al., 1971, 1975; Rutter and Morton, 1977), a Swedish model called CANOPY (Halldin et al., 1979; Perttu et al., 1980), Calder (1977), Massman (1980), and several other models described by Eriksson and Grip (1979).

Fundamental to these computer models is an expression relating the rate at which intercepted rainfall drips from the forest canopy to the amount of water stored inside the canopy. For example, Massman (1980) used the following (fairly general) expression

Drip Rate =  $D_0 \left[ \exp \left( \alpha S/S_c \right) - 1 \right] / \left[ \exp \left( \alpha \right) - 1 \right]$  (6)

where  $D_0$  and  $\alpha$  are empirical constants to be determined by optimization techniques and S is the amount of water stored inside the tree. Halldin et al. (1979) independently proposed a very similar model: Drip Rate =  $a[\exp(bS)$ -1] where a and b are empirical constants determined by optimization techniques. Often the resulting mass balance equation does not have an analytical solution and the large amount of detailed observational data makes a computer necessary. However, analytical solutions are still possible and Massman (1980) explored the requirements for obtaining them. Analytical solutions to dynamic models (termed analytical models) have many of the advantages of static models, without relying so heavily upon empiricism; further, they do not necessarily require a complex computer program or large amounts of detailed information.

In addition to needing a computer program, dynamic models, which use drip expressions similar to (6), have been criticized because the drip parameters,  $D_0$  and  $\alpha$ , are possibly site specific (Aston, 1979) and the model predictions do not always give acceptable results (Aston, 1979; Erickson and

(5)

Grip, 1979). Furthermore, the drip parameters are often determined after the rain has stopped and hence the drip expression may not be applicable during rain (Aston, 1979). The ideal model is one which maintains a sound physical basis like the dynamic models and hence minimizes empiricism, and yet which incorporates the simplicity of an analytical model. Gash's analytical model (Gash, 1979) approaches this ideal. However, his model does not deal with the drip rate, unlike the dynamic models. Therefore, Gash's model, unlike Rutter's model, does not include detailed information for the prediction of instantaneous amounts of water inside the canopy. This paper attempts to formulate a dynamic model having the general features of the ideal model, but which also includes detailed information on the instantaneous amounts of water inside the canopy.

Complementary to the process of interception is the process of evaporation, and just as there are a variety of interception models there are also a variety of evaporation models. The evaporation of intercepted rainfall from forest canopies is a complex process caused by the advection of energy from air passing over the forest, and not by radiation (Stewart, 1977; Singh and Szeicz, 1979; Pearce et al., 1980). Evaporation is turbulent and spatially inhomogenous and no present model has dealt with this process in its full complexity. Nonetheless, there are a variety of methods which have been used to model evaporation rates from forest canopies. Murphy and Knoerr (1975) devised an elaborate computer model and Brutsaert (1979) derived a sophisticated analytical model. Basic to both these models is the mixing length hypothesis with its many drawbacks (e.g., Legg and Monteith, 1975; Raupach et al., 1980). A more commonly used model is the Penman-Monteith equation which is used in conjunction with meteorological data gathered at frequent intervals. The Penman-Monteith equation is a combination of energy balance and aerodynamic methods. This paper attempts to formulate a new method for estimating evaporation rates from forest canopies, but which is easier to use than these more general models.

Thus, the specific purpose of this paper is fourfold: (1) to examine the utility of drip expressions similar to (6); (2) to demonstrate a better, less empirical model for drip rate; (3) to derive an analytical model based upon this more general dynamic model; and (4) to outline the conceptual framework for a new method of estimating evaporation rates from forest canopies. This study is a first step in a larger investigation into the influence that old-growth Douglas fir (*Pseudotsuga menziesii* [Mirb.] Franco) canopies have on the input of nitrogen to forest floors in western Oregon, U.S.A. (Carroll, 1980). During rainstorm events, nitrogen (the growth limiting nutrient in such forests — Heilman and Gessel, 1963; Miller and Fight, 1979) is leached from the canopy by intercepted rain as it drips to the forest floor. A variety of biological sources and sinks for nitrogen become active inside the canopy only when wet. Thus, the study of specific rainstorms and events is strongly motivated by a desire to eventually quantify nitrogen cycling in old-growth Douglas fir canopies.

I studied a single old-growth Douglas fir — named Minerva — which is approximately 70 m in height. The lowest branch of Minerva's canopy is about 31 m above the ground and is slightly more than 7 m long. Thus, the canopy is about 40 m in depth and about 15 m in diameter. Minerva's canopy volume is relatively free of branches from any nearby trees: < 1% of the ground projected area of Minerva's canopy overlaps the ground projected area of any other nearby tree canopy. Minerva is located in the Andrews National Forest (approximately 44°N 122°W) on the western slope of the Cascades in Oregon, U.S.A. at an elevation of about 500 m.

Horizontal wind speed was measured at the treetop with a Weathertronics micro-response cup anemometer. The air temperatures at the top and bottom of the canopy were both measured as the average of two wire wound resistance thermometers manufactured by Fenwall Electronics. At the base of the tree was a Cognition pressure transducer for determining atmospheric pressure, which was then used in estimating potential temperature and atmospheric density. The dry adiabatic lapse rate was used to estimate potential temperature. The data from these meteorological sensors were recorded every 10 min on an automated data gathering system. In the case of the anemometer, the total number of revolutions in every 10-min sampling interval was recorded and so the anemometer was not simply spot checked every 10 min. All sensor readings were then averaged over the duration of the rainfall event.

The treetop sensors were mounted on a 20' vertical spar which was attached to the trunk of the tree. The sensors at the bottom of the canopy were mounted on a 20' horizontal spar located near the bottom branch. The temperature sensors were painted white and shielded from direct radiation by the way they were mounted on the spar. Obviously these measurements are prone to errors. For example, the movement of the treetop may cause errors in the wind speed. Mounting sensors on a tower would eliminate this problem, but for this study a tower was impractical and expensive. Sampling errors in the temperature measurements can also arise due to the horizontal variability of the temperature field, especially with such a large canopy. However, during rainfall events, the variability in the temperature field is probably not as great when it is wet, as when it is dry.

Gross precipitation and throughfall were collected in troughs designed after Best (1976). They are 2 m long, 0.15 m wide and placed slightly more than 1 m above the ground. The exact height varied somewhat with the terrain. The troughs sloped slightly to one end where they were connected by plastic tubing to tipping buckets which rest on leveled concrete blocks. The bucket volumes were calibrated to be  $10 \text{ cm}^3 \pm 0.03 \text{ cm}^3$ . Without the collecting troughs the resolution of the tipping bucket gauge is 2.54 mm, and with the trough it is 1/30 mm. There were ten such troughs under the tree and four in a nearby clear-cut. The exact number of troughs used for precipitation collections varied from one storm to the next. In earlier storms only one clear-cut trough was operative because the others were not in place. During some storms selected clear-cut and throughfall troughs were used for precipitation chemistry experiments and were disconnected from the data logger. Data from two of the clear-cut troughs were lost for a short period when one of the cables was accidentally cut.

The number of tips for each tipping bucket were accumulated continuously and recorded electronically on digital tape every 10 min by the data logger. Gross precipitation amounts and rates were averaged over the troughs under the tree. The resolution of the system as a whole is about  $0.1 \text{ mm h}^{-1}$  for a 10-min rate measurement and about  $\pm 0.01 \text{ mm}$  for total amounts. Wetting amounts for the troughs were estimated to be < 0.1 mm.

A total of 43 storms were observed and recorded from November 1980 to December 1981. Typical rainfall rates measured with this system are  $\sim 2 \,\mathrm{mm}\,\mathrm{h}^{-1}$ , with brief and very rare extreme values of  $\sim 15 \,\mathrm{mm}\,\mathrm{h}^{-1}$ . The corresponding throughfall rates are about two-thirds of these rates. For old-growth Douglas fir trees stemflow is insignificant (Rothacher, 1963) and can be ignored.

Tipping bucket gauges are quite simple, reliable and very accurate for low to intermediate rates. However, they are known to underestimate the precipitation amounts at higher rates because of splash during the movement of the bucket (Marsalek, 1981). For this study, even with the enlarged orifice, the precipitation rates are probably accurate to within a 2% error for all rainfall rates  $< 10 \text{ mm h}^{-1}$ . As the rainfall rate decreases so does the error, so that at  $2 \text{ mm h}^{-1}$  the error is probably < 0.5%. These estimates were based on an expression derived by Marsalek (1981) for estimating the splash error. To employ this method the tipping time (the time required for one bucket to begin moving until it tipped over and came to rest) was measured and found to be very short,  $\sim 0.25$  s.

Tipping buckets also change calibration with time due to surface tension of the water passing through the bucket, the drainage of residues from the lip of the buckets and for a variety of other reasons (Marsalek, 1981). By periodically recalibrating buckets in the laboratory, the calibrations were found to change by 10-20% for individual buckets. Except for one or two tipping buckets, this was a systematic error and the rates and amounts of precipitation were always underestimated. Errors can also occur due to horizontal winds blowing rain droplets past the collecting troughs. However, for this study, these wind errors are not considered because of the very low wind speeds ( $\leq 0.4 \text{ ms}^{-1}$  at the clear-cut and under the tree).

To examine the influence that the splash and calibration errors caused, all storms were reanalyzed with another calibration value derived from the laboratory recalibration results and Marsalek's (1981) splash equation. It was found that all the major conclusions of this work held, although some slight changes occurred in specific parameter values for the models. The major effect was that the recalibrated data predicted amounts in the tree that exceeded those amounts predicted without the recalibration by 0.1-0.4 mm. This

translates into a 10–15% difference. The exact calibration error for any given storm is not known; furthermore, it probably changes with time and the nature of the storm. Thus, I will use the original data without recalibration, acknowledging a possible error of  $\sim 15\%$ , and outline, where appropriate, how these errors influence the results.

The rest of this paper is divided into five sections. The first section describes attempts to validate the drip expression given by (6) with data collected by the system for this study. The next two sections outline the new interception model: first in its dynamic form and then in its analytical form. The fourth section derives a new method for estimating evaporation rates and the fifth section contains concluding remarks.

# DATA ANALYSIS

The initial step in the analysis was to determine the canopy capacity and the free throughfall parameter. The saturation amount is inferred from plots of total throughfall (denoted  $P_N$ ) vs.  $P_G$  for all storms which completely saturate the (initially dry) tree (Leyton et al., 1967). Figure 1 is a plot of  $P_N$  vs.  $P_G$  for all such storms recorded. The straight line for the saturated storage gives a value of 1.5 mm for  $S_c$  (the saturation amount). The recalibrated data suggest a value of 1.6 mm. Obviously, such a method is crude and prone to errors which arise because evaporation is not considered. Furthermore, the saturation amount may vary from one storm to another according to a variety of factors: wind, temperature, and size and momentum of rain droplets (Leonard, 1967; Jackson, 1975). However, direct observational evidence by Hancock and Crowther (1979) does validate the concept of a saturation amount.

The value of 1.5 mm for  $S_c$  was checked by an independent method based upon a detailed description of the amount and type of foliage and epiphytes contained in the tree canopy. From laboratory misting studies of needles, twigs and epiphytes (Perkins and Carroll, University of Oregon, unpublished data), and a first round tree description of Minerva (e.g., Pike et al., 1977), the total estimated water holding capacity of Minerva's canopy is about 300 kg of water or  $S_c = 1.75 \pm 0.4$  mm when distributed over the ground projected area of Minerva's canopy. This estimate for  $S_c$  probably represents a maximum value, because the percentage of foliage surfaces covered by water is likely to be higher in laboratory misting experiments than during rainfall.

Like  $S_c$ , the free throughfall parameter, p, is usually estimated from plots of  $P_N$  vs.  $P_G$ , except that only those storms which do not saturate the tree are used (Rutter et al., 1971). However, this method could not be used, since only one such storm occurred. Therefore, p was estimated by computing the ratio of throughfall and rain rates for each storm during the time between when the storm began and when the first measurable amounts of throughfall were recorded. The value obtained was  $p = 0.05 \pm 0.03$ . For a canopy of



Fig. 1. Gross precipitation,  $P_G$ , vs. total throughfall,  $P_N$ , for storms between November 21, 1980 and December 10, 1981 for a single old-growth Douglas fir tree. The straight line gives an approximate value of 1.5 mm for the saturation amount of this tree.

such depth this method may work well because intercepted throughfall should be delayed, however, a data sampling interval shorter than 10 min, as used here, would probably give a more reliable result. Recalibrating the data had a negligible effect on this value of p. Aston (1979) suggests that p can be estimated from the leaf area index (*LAI*). His relationship, given approximately as: p = 1.0-0.05 (*LAI*), combined with a value of 19.2 for total needle surface index for Minerva (Massman, 1982), predicts p = 0.04. Such close agreement between the two methods is encouraging, but it may be coincidental.

The amount of water inside the canopy during each 10-min sampling interval, denoted S, was estimated by a running balance between rain, throughfall and evaporation. The evaporation rate was parameterized after Rutter et al. (1971) as follows

$$E(t) = \begin{cases} E_0 S/S_c & \text{when } S \leq S_c \\ E_0 & \text{when } S \geq S_c \end{cases}$$
(7)

where E(t) is the evaporation rate as a function of time, and  $E_0$  is a constant evaporation rate for the period between when the rain begins and when the drip ends. The assumption that evaporation rates are proportional to  $S/S_c$ whenever  $S \leq S_c$  was made arbitrarily by Rutter et al. (1971). However, since that time Hancock and Crowther (1979) have given it observational verification and Shuttleworth (1978) has shown that it leads to a theoretically reasonable description of an unsaturated wet canopy. However, eq. 7 is not the only way of parameterizing E(t). More recently, Sellers and Lockwood (1981) have proposed that  $E(t) = E_0 [a(S/S_c)^2 + c(S/S_c) + q]$  is more realistic for the evaporation rate whenever  $S \leq S_c$ . This latter expression has fostered some debate (Shuttleworth and Gash, 1982) and is still under investigation (Sellers and Lockwood, 1982). Thus, it seems premature to use Sellers and Lockwood's evaporation model at this time.

The evaporation rate,  $E_0$ , was computed numerically using (7) and a computer program. The program began with an initial value of  $E_0$  and then estimates S for every 10-min data interval from the time the rain began until the tree stopped dripping. If the amount left in the tree at the end of this time was not equal to  $S_c \pm 0.005 \,\mathrm{mm}$ , then  $E_0$  was adjusted and the computation re-done. This procedure was repeated until the final value of S converged to  $S_c \pm 0.005 \,\mathrm{mm}$ .

Obviously, this approach assumes that  $E_0$  is constant throughout the storm. Thus,  $E_0$  does not include diurnal variations in the evaporation rate. However, because advection of energy drives evaporation rather than radiation (Singh and Szeicz, 1979; Stewart, 1977; Pearce et al., 1980), the diurnal variations may not be significant. Furthermore, Pearce et al. (1980) found that night-time evaporation rates are equal to daytime evaporation rates during rainfall periods for at least one evergreen mixed forest. Therefore, the assumption that  $E_0$  is constant during the length of a rainstorm seems reasonable even at this field site, where the rain storms are often about 24 h long.

The next step in the analysis was to validate the drip model given by (6) by plotting these estimated values of S vs. their corresponding values of the observed drip rate, D(t). The drip rate, D(t) is the total throughfall rate minus the direct throughfall rate. A non-linear regression program was used to compute optimal values of  $D_0$  and  $\alpha$  from (6) by a least squares optimization technique. Figures 2 and 3 are examples of the data and curve fits for the February 23 and June 7, 1981 storms, respectively. Figure 2 shows one of the better fits and Fig. 3 one of the poorer fits. The results of this analysis and other pertinent data for each of the 20 test storms are tabulated in Table I. The recalibrated data show very similar results for  $D_0$  and  $\alpha$  values. The  $R^2$  for each fit was computed according to the following formula:  $R^2 = 1 - SSQR/SSQT$  where SSQR is the residual sum of squares of the regression curve and SSQT is the original variance of the data.

The results as shown in Table I are rather unsatisfactory. The parameter values  $D_0$  and  $\alpha$  vary considerably from one storm to the next and the quality of the fit  $(R^2)$ , range from very good to very poor. It is possible that wind has some influence on  $\alpha$  (Perttu et al., 1980; Massman, 1980) which might account for its large variability. However, this explanation is not entirely convincing. Winds are usually very light during storms at this site typically  $\leq 1 \text{ ms}^{-1}$  at the treetop). Furthermore, Perttu et al. (1980) found that the drip rate was only weakly dependent upon wind speed even for relatively high wind speeds (up to  $7 \text{ ms}^{-1}$ ).

Another possible explanation for these unsatisfactory results is that drip expressions similar to (6) are not valid during rainfall. Both Rutter et al.



Fig. 2. The observed drip rate of intercepted rainfall vs. the estimated amount of water held inside the canopy of a single old-growth Douglas fir tree, S, for every 10-min interval during the February 23, 1981 storm. The curve gives the best fit of the observed data with the drip expression given by eq. 6. The  $R^2$  of this fit is 0.58.



Fig. 3. The observed drip rate of intercepted rainfall vs. the estimated amount of water held inside the canopy of a single old-growth Douglas fir tree, S, for every 10-min interval during the June 7, 1981 storm. The curve gives the best fit of the observed data with the drip expression given by eq. 6. The  $R^2$  of this fit is 0.13.

# TABLE I

Observed and modelled storm characteristics with Rutter-like drip model (eq. 6) and new drip model (eq. 8)

| Storm date | P <sub>G</sub><br>(mm) | P <sub>N</sub><br>(mm) | Rainfall<br>duration<br>(h) | $\frac{R_0}{(\mathrm{mm}\ \mathrm{h}^{-1})}$ | Drip duration<br>after rainfall<br>has ceased<br>(h) | $\frac{E_0}{(\mathrm{mm}\ \mathrm{h}^{-1})}$ | Eq. 6<br>$D_0$<br>(mm h <sup>-1</sup> ) | Eq. 6<br>α | Optimal predictions of $P_N$ (eq. 6) (mr | R <sup>2</sup><br>; (eq. 6)<br>n) | R <sup>2</sup><br>(eq. 8) |
|------------|------------------------|------------------------|-----------------------------|--|--|--|---|------------|--|-----------------------------------|---------------------------|
| Feb. 23    | 14.83                  | 11.67                  | 9.67                        | 1.53   | 4.33   | 0.12   | 0.16                                    | 1.55       | 12.16                                    | 0.58                              | 0.83                      |
| Mar. 3     | 13.53                  | 10.32                  | 19.33                       | 0.70   | 3.17   | 0.08   | 0.17                                    | 2.21       | 9.83                                     | 0.20                              | 0.54                      |
| Mar. 7     | 2.30                   | 0.82                   | 5.67                        | 0.41   | 1.50   | 0.00   | 0.16                                    | -0.72      | 0.88                                     | 0.09                              | 0.58                      |
| Mar. 15    | 30.53                  | 25.08                  | 31.33                       | 0.97   | 0.33   | 0.13   | 0.31                                    | 1.32       | 24.58                                    | 0.30                              | 0.61                      |
| Mar. 19    | 5.07                   | 2.90                   | 7.00                        | 0.72   | 0.67   | 0.11   | 0.31                                    | 2.69       | 2.64                                     | 0.21                              | 0.63                      |
| Mar. 21    | 22.67                  | 19.08                  | 23.67                       | 0.96   | 0.50   | 0.10   | 0.28                                    | 1.86       | 18.32                                    | 0.39                              | 0.79                      |
| Mar. 24    | 23.80                  | 20.98                  | 17.17                       | 1.39   | 1.00   | 0.09   | 0.65                                    | 1.41       | 21.26                                    | 0.76                              | 0.87                      |
| Mar. 28    | 59.27                  | 50.85                  | 57.67                       | 1.03   | 3.00   | 0.12   | 0.48                                    | 0.67       | 50.66                                    | 0.04                              | 0.55                      |
| Apr. 15    | 2.77                   | 1.13                   | 8.00                        | 0.35   | 3.33   | 0.02   | 0.13                                    | -3.61      | 1.25                                     | 0.14                              | 0.69                      |
| Apr. 20    | 4.50                   | 2.60                   | 11.33                       | 0.40   | 1.50   | 0.04   | 0.19                                    | -7.08      | 2.61                                     | 0.02                              | 0.52                      |
| May 14     | 28.13                  | 18.32                  | 38.00                       | 0.74   | 3.67   | 0.22   | 0.37                                    | 0.98       | 18.88                                    | 0.03                              | 0.62                      |
| May 23     | 29.90                  | 25.52                  | 39.00                       | 0.77   | 1.83   | 0.10   | 0.56                                    | 0.89       | 25.13                                    | 0.73                              | 0.80                      |
| June 5     | 9.07                   | 3.42                   | 10.50                       | 0.86   | 1.67   | 0.43   | 0.024                                   | 4.12       | 2.59                                     | 0.42                              | 0.79                      |
| June 7     | 106.05                 | 79.42                  | 57.67                       | 1.84   | 1.00   | 0.45   | 0.70                                    | 0.42       | 81.94                                    | 0.13                              | 0.81                      |
| July 6     | 17.67                  | 12.92                  | 25.17                       | 0.70   | 1.17   | 0.15   | 0.14                                    | 1.93       | 13.66                                    | 0.65                              | 0.69                      |
| Sep. 18    | 9.93                   | 6.70                   | 9.00                        | 1.10   | 1.83   | 0.25   | 0.096                                   | 2.98       | 6.90                                     | 0.80                              | 0.93                      |
| Oct. 2     | 7.70                   | 5.13                   | 4.00                        | 1.93   | 2.33   | 0.18   | 0.16                                    | 4.21       | 4.81                                     | 0.54                              | 0.85                      |
| Nov. 11    | 31.92                  | 25.28                  | 20.00                       | 1.60   | 1.17   | 0.25   | 0.49                                    | 0.93       | 25.07                                    | 0.25                              | 0.81                      |
| Dec. 5     | 139.29                 | 136.16                 | 53.50                       | 2.71   | 3.00   | 0.03   |   |            | -  | _                                 | _                         |
| Dec. 9     | 18.82                  | 16.55                  | 22.17                       | 0.87   | 2.00   | 0.03   | _                                       | —          |  | -                                 | -                         |

271

(1971) and Aston (1979) fit their drip expressions with data taken only after the rain had stopped and their resulting parameter values presumably showed much less variability, however, no mention is made in either paper of the quality of the fit or the variability in the drip data. By direct observation, Aston (1979) found that the Rutter drip expression allowed S to be overestimated during periods of rainfall. Furthermore, the results of this study show that the use of any single pair of optimal values of  $D_0$  and  $\alpha$  could introduce substantial and unacceptable errors into predictions of throughfall amounts for any given storm and, eventually, for yearly totals. Eriksson and Grip (1979) concluded that the Swedish model, CANOPY, which uses drip expression similar to (6) and was validated in a similar manner to that used in this work (i.e., including drip data when rain was falling, Perttu et al., 1980), lead to unacceptably high values for interception loss. They, like Aston, found that the predicted amounts of water retained by the canopy were too high. Thus, to employ expressions similar to (6), it may be more realistic to choose parameter values on a seasonal basis rather than using individual storms and this approach is explored in a later section.

# A NEW APPROACH

Since the results in the last section were unacceptable, a new approach was tried. Massman (1980) suggested that the drip parameters  $D_0$  and  $\alpha$  may also be influenced by the rain rate; this section explores the possibilities of incorporating the rain rate directly into drip expressions similar to (6).

In addition to scatter diagrams of drip rate, D(t), vs. S, as shown in Figs. 2 and 3, drip rates vs. rain rates for each storm were also plotted. Figure 4 is the scatter diagram of D(t) vs. R(t) for the February 23, 1981 storm. Only those data which were recorded between the times when the rain began and when it ended are plotted. There are 58 such data points for the February 23 storm. However, because of duplication of certain points this plot appears to have only 48 points. The correlation coefficient between drip rate and rain rate in this figure is 0.75, which is significant to a level > 99.95%. In fact, the correlation coefficients between drip and rain rates were significant at levels > 99.95% for all storms tested. Furthermore, the drip rate was more strongly correlated to rain rate than to S, which suggests that the rain rate should be explicitly modeled into the drip rate expression.

One possibility which might explain the high correlation between rain and drip rates is that the free throughfall coefficient, p, was underestimated; however, this does not appear to be the case. Old-growth Douglas fir trees tend to have extremely closed canopies and a high value for p is to be expected. Even so, the correlation coefficients were recomputed assuming a value of 0.09 for p and the results were virtually the same. The p value would have to be increased to somewhere between 0.25 and 0.50 before it could begin to account for such high correlations. Such a p value is impossibly large. It is also possible that p is a function of rainfall rates and other storm variables. Aston (1979) found that p increased as the rain rate



Fig. 4. A scatter diagram of observed drip rates of intercepted rainfall vs. the observed rainfall rates for every 10-min interval during the February 23, 1981 storm for a single old-growth Douglas fir tree. The correlation coefficient between these drip and rainfall rates is 0.75 which is significant at the 99.95% confidence level.

decreased. However, our data does not show any significant variation in p either with rain rate or season of the year. It is worth noting, however, that rain rates at our study site tend to be much lower than those simulated by Aston (1979).

The drip model which was tried next is a simple expansion of (6) and is given as

$$D(t) = (D_0 + d_0 R(t)) \{ [\exp(\alpha S/S_c) - 1] / [\exp(\alpha) - 1] \}$$
(8)

where  $d_0$  is a model parameter. Conceptually, it is more realistic to use a term proportional to the interception rate such as  $d_0(1-p) R(t)$  rather than  $d_0R(t)$ . However, for our purposes  $d_0$  will just subsume the (1-p) factor. The drip parameters,  $D_0$ ,  $\alpha$ , and  $d_0$ , were determined for each of the 18 test storms using non-linear optimization techniques in a manner similar to that employed with (6), except that now the rain rate is explicitly included. The  $R^2$  values for this new model are shown in the last column of Table I and there is substantial improvement in the quality of the fit in all storms. The  $R^2$  values of this new drip model are often double or triple the  $R^2$  values for the original model. Thus, much of the observed variability in the drip rate is directly attributable to the variability in the rain rate. This suggests that the drip rate is controlled in some manner by the interaction of rain rate and storage inside the canopy.

The specific parameter values corresponding to (8) are not shown

because it is only an intermediate model. The model (8) is presented here only as motivation for the later model and to show that there is a connection between the original model, (6), and the model to be discussed later. The parameter values corresponding to (8) did vary from storm to storm; however, both  $D_0$  and, especially  $\alpha$ , showed less variability than with the original model. The variability in the parameters was still too high to be completely satisfactory. However, the values of the parameters  $\alpha$  did suggest that the model could be simplified by replacing [exp ( $\alpha S/S_c$ ) - 1]/[exp ( $\alpha$ ) - 1] with  $S/S_c$  (equivalent to  $\alpha = 0$ ). With this simplification and further testing, the following model was formulated

$$D(t) = [D_0 + d_0 R(t)] [S/S_c]$$
(9)

where  $D_0$  has been fixed at a value of  $0.12 \text{ mm h}^{-1}$  and  $d_0$  is determined for each storm by the following equation

$$d_{0} = (P_{N} - pP_{G} - D_{0}T_{1}\bar{S}/S_{c}) / \{ \int_{0}^{T_{1}} [R(t)S/S_{c}] dt \}$$
(10)

where  $T_1$  is the length of time between when the storm begins and when the canopy drip ceases; and  $\overline{S}$  is the average amount of water inside the tree canopy during the  $T_1$  time period. A derivation of (10) is given in the appendix. Choosing  $d_0$  in this manner guarantees that the model predictions for gross interception loss will coincide exactly with the observed values of  $P_G - P_N$ . The results from this model are tabulated in Table II. Use of the recalibrated data showed little effect upon  $d_0$  or the  $\mathbb{R}^2$  values. Besides the 18 test storms already discussed, Table II also shows the results of fits for two other storms which were chosen to independently verify the results of the other, earlier storms. For these two storms, the recalibrated data were used. These are expected to be quite accurate because the tipping buckets were recalibrated shortly after the storms were recorded. The  $R^2$  values for these last two storms are 0.89 and 0.75, respectively. A comparison of the  $R^2$  values between the original drip model, (6), and this simpler model, (9), shows that the fits with the later model are significantly better than with the original model and with essentially no empirically determined constants.

The exact numerical value of  $D_0$  had remarkably little effect on the quality of any individual fit providing  $d_0$  was adjusted according to (10). Thus,  $D_0$  can be reasonably chosen as the minimum drip rate which the data gathering system is capable of detecting, as Rutter et al. (1971) originally proposed. The Rutter model seems inconsistent on this point because in a later version of the model (Rutter et al., 1975) it is suggested that  $D_0$  is site specific and depends upon *LAI*. Aston (1979) likewise criticizes the Rutter drip model for being site specific. A value of 0.12 mm h<sup>-1</sup> for  $D_0$  was judged to be close enough to the theoretical resolution of the system in this study  $(0.10 \text{ mm h}^{-1})$  to be satisfactory. Results using  $D_0 = 0.10 \text{ mm h}^{-1}$  gave slightly poorer results.

The exact value of  $d_0$  is obviously dependent upon individual storm characteristics. It does vary from one storm to another and is neither simple

| TA | BI | $\mathbf{E}$ | Π |
|----|----|--------------|---|
|    |    |              |   |

 $d_0$  and  $R^2$  values for drip model given by eqs. 9 and 10

| Storm date | d <sub>0</sub> | $R^2$ |
|------------|----------------|-------|
| Feb. 23    | 0.27           | 0.75  |
| Mar. 3     | 0.36           | 0.46  |
| Mar. 7     | 0.14           | 0.42  |
| Mar. 15    | 0.38           | 0.40  |
| Mar. 19    | 0.41           | 0.56  |
| Mar. 21    | 0.42           | 0.58  |
| Mar. 24    | 0.48           | 0.75  |
| Mar. 28    | 0.18           | 0.28  |
| Apr. 15    | 0.04           | 0.25  |
| Apr. 20    | 0.25           | 0.41  |
| May 14     | 0.23           | 0.46  |
| May 23     | 0.50           | 0.55  |
| June 5     | 0.12           | 0.74  |
| June 7     | 0.16           | 0.75  |
| July 6     | 0.26           | 0.54  |
| Sep. 18    | 0.28           | 0.60  |
| Oct. 2     | 0.40           | 0.80  |
| Nov. 11    | 0.37           | 0.75  |
| Dec. 5     | 0.11           | 0.89  |
| Dec. 9     | 0.33           | 0.75  |

nor constant. Without the aid of a computer program and some extensive instrumentation, accurate estimates of  $d_0$  for use in a dynamic model at the level of individual storms may be quite difficult. However, accurate estimates of  $d_0$  at the level of individual storms may not be entirely impossible. Detailed studies of whether  $\overline{S}$  and the interaction term,  $\int_0^{T_1} [R(t)S/S_c] dt$ , are related to more easily observed storm characteristics must be made first. Another possible approach would be to estimate a single value of  $d_0$  by adjusting it so that the total gross interception loss over an entire season or year could be accurately simulated. However, this latter approach would probably not predict individual storms very well and may be dependent upon the data set used to validate the approach. Thus, this latter approach may not be easily generalized to other forests. For the purposes of this work these possibilities will not be explored. However, there is an alternative approach worth exploring. It is outlined in the following section, where the dynamic model will be simplified to an analytical (or static) model. In the analytical model, the numerical complexities introduced by (10) are virtually eliminated.

The basic purpose of this and the preceding section has been to establish that the drip expressions similar to (6) are inadequate. The failure of these drip expressions (at least for this study) is probably due to the fact that they assume that the drip rate is determined solely by the amount of water stored in the tree canopy and they do not explicitly account for the influence of the rain rate. Furthermore, I suspect that problems incurred in using drip expressions similar to (6) which other researchers have had, i.e., Aston (1979), Eriksson and Grip (1979), Perttu et al. (1980), are caused for the same reason: they do not explicitly account for the influence of the rain rate upon the drip rate.

I further propose that there is a basic physical effect (the dislodgement of previously intercepted rain droplets by falling rain droplets) which is better described by models similar to (8) and (9) than by (6). Thus, the following process is envisioned: after some critical amount of water has been intercepted by foliage surfaces near the top of the tree further interception at that level would dislodge some portion of the stationary droplets which would then fall out of the tree or be intercepted at lower levels. At lower levels the process would be continued. Calder (1977) emulated this cascade process with a model of a layered canopy and found that his model gave better predictions. Thus, during rainfall the drip may be controlled by a cascade of water droplets falling through the tree as they alternately fall and impact upon lower surfaces. Such a cascade process would probably be strongly regulated by the momentum of the rain droplets just above the canopy. Jackson (1975) also reasoned that the momentum of raindrops could have an influence upon storage and drainage. One gross measure of this momentum is the rain rate. It is this cascade effect which may give rise to interaction terms related to  $R(t) S/S_c$  in models (8) and (9).

Whether cascade models similar to (8) and (9) can improve predictions in other forests is difficult to tell without further study. Other factors which may be important to consider are the type of foliage and canopy structure. This process may be important in forests which have large amounts of foliage near the top of the canopy as is the case with old-growth Douglas fir trees (Massman, 1982). It may also be important in areas where high rain rates are frequent; however, it is noteworthy that even for the low rain rates reported here the cascade process would appear to be very important.

This concludes the discussion on the dynamic model. The next section derives an analytical model from the dynamic model discussed in this section. The analytical model contains much of the physical reasoning behind the dynamic model, but it is much easier to use, much less sensitive to the exact numerical values used in the dynamic model and still gives quite accurate results for each storm.

#### ANALYTICAL MODEL

By reformulating the dynamic model using a constant rain rate,  $R_0$ , in place of a variable rain rate, R(t), the dynamic model can be integrated exactly to give an analytic model which predicts gross interception loss. Thus, the dynamic model is given as follows

$$\frac{dS}{dt} = \begin{cases} (1-p)R_0 - [D_0 + d_0R_0]S/S_c - E_0S/S_c & \text{when } S \leq S_c \\ (1-p)R_0 - [D_0 + d_0R_0]S/S_c - E_0 & \text{when } S \geq S_c \end{cases}$$
(11A)

the gross interception loss is defined as  $P_G - P_N = S_{evap} + S_c$  where  $S_{evap}$  is the amount of water evaporated between when the storm begins and the drip from the canopy ceases. I shall assume that the storm is sufficient to wet the canopy in a finite time,  $t_1$ . Therefore,  $S_{evap}$  is given as follows

$$S_{\text{evap}} = \int_{0}^{t_{1}} (E_{0} S/S_{c})dt + E_{0} (T_{1} - t_{1})^{2}$$
(12)

From (11A) it follows that

t1

$$S/S_{c} = \{1 - \exp[-A(1 - p)R_{0}t/S_{c}]\}/A$$
(13)

and

$$t_1 = -S_c \ln (1-A) / [(1-p) R_0 A]$$
(14)

where  $A = (D_0 + d_0R_0 + E_0)/[(1-p)R_0]$ . Thus, the assumption that the storm be sufficient to saturate the tree is equivalent to the inequality: A < 1, and hence  $t_1$  is finite. Substituting (13) and (14) into (12), the interception loss is given as

$$I = S_{\text{evap}} + S_c = S_c \{ 1 - \beta E_0 / [(1 - p)R_0] \} + E_0 T_1$$
(15)

where  $\beta = [A + (1 - A) \ln (1 - A)]/A^2$ . Thus, the specific drip dynamics have been incorporated into  $\beta$ . However, because  $\beta$  obeys the following inequality:  $\frac{1}{2} \leq \beta \leq 1$ , the specific drip dynamics are far less important than in the original dynamic model. A proof of the inequality is fairly simple and is given in the appendix. It should be re-emphasized that (15) is valid only when A < 1, which requires that  $E_0 < (1 - p)R_0$ . In the event that the evaporation rate,  $E_0$ , exceeds the interception rate, then A > 1 and the tree will never reach complete saturation.

This analytical expression for I is rather simple. It incorporates many of the more empirical models described earlier, but it is based on more fundamental physical reasoning. Furthermore, (15) predicts that the gross interception loss will increase as the rain rate increases, which agrees with the observations of Jackson (1975), Schulze et al. (1978), and Aston (1979). Proof of this prediction follows from establishing that  $\partial I/\partial R_0 > 0$  for all values of  $R_0$  and is also given in the appendix. To test whether I increases as  $R_0$  increases the correlation coefficient, r, between the observed interception loss  $(P_G - P_N)$  and the mean rain rate  $(R_0)$ , was computed for the data presented in this study. The results showed that r was positive, but not statistically significant. Thus, the data presented here do not clearly substantiate that I is positively correlated to  $R_0$ .

The parameter  $\beta$  does vary from one storm to the next, however, the range of values it takes on is quite small. For the storm data in this study: 0.57

 $\leq \beta \leq 0.77$ . To test the utility of (15),  $\beta$  was fixed at a value of 0.75 (the data suggest that  $\beta = 0.67$  would be more appropriate, but 0.75 was chosen because it is the average of 0.50 and 1) and the observed and predicted gross interception losses were compared for several of the storms. The results are shown in Table III. The first column of this table gives the observed gross interception loss with the 95% confidence limits in parentheses for each storm tested. The second column gives the predictions for *I* with the new model (eq. 15 with  $\beta = 0.75$ ). The third column gives the predictions for *I* using the exponential drip rate model (eq. 6 with  $D_0 = 0.10 \text{ mm h}^{-1}$  and  $\alpha = 1.40$ ).

For the new model, eq. 15, the total predicted gross interception loss for all storms is ~ 4% above the observed loss. Most of the overestimate (~ 90% of it) is associated with the 4 summer storms (June 5–September 18), when evaporation rates were high. This suggests that (15) is quite sensitive to the evaporation rate and underscores the need for an accurate estimate of  $E_0$ . The recalibrated data showed a 3% overestimation of the observed data.

Further study may refine the choice of  $\beta$  for these forests. Extension of (15) to other forests is, of course, possible; however, the parameter  $\beta$  may change with the type of foliage and other rainfall characteristics. More accurate results could probably be obtained by specific incorporation of the change in  $\beta$  as rain and evaporation rates change for individual storms. However, our results with a fixed value for  $\beta$  seem quite good.

For the exponential drip model, eq. 6, a variety of  $D_0$  and  $\alpha$  values ranging from  $D_0 = 0.10$  and  $\alpha = 1.20$  to  $D_0 = 0.20$  and  $\alpha = 2.00$  were tried. The total predicted *I* for the 18 storms tested varied from 4% above the observed total to 6% below it for these  $D_0$  and  $\alpha$  values. The predictions of (6) shown in Table III used  $D_0 = 0.10$  and  $\alpha = 1.40$ . For this choice of parameter values (6) predicted an amount for the total *I* which was 3% above the observed amount. Furthermore, this choice of parameter values was judged to be the best of all those tested because the amount of water remaining in the canopy when the drip ceased,  $S_f$ , was within 0.02 mm of  $S_c$  for every storm. The other parameter values produced a much broader range of values for  $S_f$  and in some cases  $S_f$  differed from  $S_c$  by 0.50 mm.

Unlike the earlier results (cf. Table I) where the ability of eq. 6 to predict the instantaneous drip rate was judged inadequate, the present results, which show its ability to predict I for individual storms and yearly totals, appear to be acceptable. Furthermore, eq. 6 does have the advantage of relative insensitivity to the exact choice of parameter values (to within a certain limited range). However, eq. 6 has two important disadvantages.

First, the best choice of parameter values was mostly found by trial and error using the entire data set, and earlier attempts, using non-linear regression techniques on a per storm basis, were virtually useless. Hence the values of  $D_0 = 0.10$  and  $\alpha = 1.40$  may be valid for this specific data set and not necessarily valid for any other data det. Second, the best choice of  $D_0$ and  $\alpha$  values does not resemble any of the optimal values found for individual

Comparison of observed and predicted gross interception loss

| Storm date | Observed gross<br>interception loss<br>$P_G - P_N (\pm 95\%)$<br>confidence limits) | Predicted gross<br>interception loss<br>eq. 15 | Predicted gross<br>interception loss<br>eq. 6 with<br>$D_0 = 0.10 \text{ mm h}^{-1}$<br>$\alpha = 1.4$ |  |
|------------|---|--|--|--|
|            | (mm)  | (mm)   | (mm)   |  |
| Feb. 23    | 3.16 (± 6.34)   | 3.09   | 2.99   |  |
| Mar. 3     | $3.21(\pm 5.02)$  | 3.25   | 3.25   |  |
| Mar. 7     | 1.48 (± 0.72)   | 1.50   | 1.53   |  |
| Mar. 15    | 5.45 (± 8.50)   | 5.46   | 5.96   |  |
| Mar. 19    | $2.17(\pm 1.58)$  | 2.18   | 2.48   |  |
| Mar. 21    | $3.79(\pm 12.16)$   | 3.70   | 3.81   |  |
| Mar. 24    | $2.82(\pm 11.74)$   | 2.94   | 3.31   |  |
| Mar. 28    | $8.24(\pm 15.17)$   | 8.41   | 8.65   |  |
| Apr. 15    | 1.64 (± 1.01)   | 1.65   | 1.64   |  |
| Apr. 20    | 1.90 (± 2.58)   | 1.86   | 1.92   |  |
| May 14     | 9.81 (± 9.40)   | 10.15  | 9.24   |  |
| May 23     | $4.38(\pm 14.42)$   | 5.52   | 5.89   |  |
| June 5     | $5.65(\pm 2.34)$  | 6.14   | 5.85   |  |
| June 7     | 26.63 (± 25.85)   | 27.79  | 25.62  |  |
| July 6     | 4.75 (± 4.59)   | 5.20   | 4.80   |  |
| Sep. 18    | $3.23(\pm 1.34)$  | 3.94   | 3.56   |  |
| Oct. 2     | 2.57 (± 1.04)   | 2.53   | 2.90   |  |
| Nov. 11    | 6.64 (± 9.74)   | 6.61   | 7.23   |  |
| Dec. 5     | 3.17 (± 67.03)  | 3.19   | <u> </u>   |  |
| Dec. 7     | 2.27 (± 2.89)   | 2.30   |  |  |

storms. Hence, predictions of the instantaneous drip rates, D(t), and the instantaneous amount of water in the canopy, S, for any single storm can be greatly in error. Furthermore, the use of a simple pair of optimal values of  $D_0$  and  $\alpha$  as listed in Table I introduced significant and unacceptable errors in model predictions for I on the level of individual storms and for the accumulated I for all storms. The exponential drip model agreed, to within 10% of the observed total I, with only 7 pairs of the optimal  $D_0$  and  $\alpha$  values; while (6) systematically underestimated or overestimated I for individual storms and differed from the total I anywhere between 20 and 100% when used with 7 other pairs of the optimal  $D_0$  and  $\alpha$  values. These results suggest that  $D_0$  and  $\alpha$  values for the exponential drip model are probably better determined on a seasonal or yearly basis than at the level of individual storms.

This completes the discussion of the interception process. The next section outlines a new approach to estimating evaporation rates from forest canopies which is based on mixing length theory, but which is much simpler to use than the more general models discussed earlier.

# EVAPORATION

With mixing length theory, turbulent heat transfer inside plant canopies is expressed as the one-dimensional steady state diffusion equation

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(K_H \frac{\mathrm{d}\psi}{\mathrm{d}z}\right) = -a(z)C_H u(z) \left(\psi_f - \psi\right) \tag{16}$$

where  $\psi$  is the concentration of sensible heat and equals  $\rho C_p \theta$ :  $\rho$  is the density of the atmosphere inside the canopy;  $C_p$  is the specific heat capacity of air at constant pressure  $(10^{-3} \text{ Jkg}^{-1} \text{ K}^{-1})$ ;  $\theta$  is potential temperature. The height above the ground is denoted by z;  $K_H$  is the thermal diffusivity; a(z) is the foliage surface area density as a function of height;  $C_H$  is the transfer coefficient and is determined by the Reynolds and Prandtl number (Brutsaert, 1979); u(z) is the horizontal wind speed profile inside the canopy; and  $\psi_f$  refers to the foliage sensible heat concentration.

The thermal diffusivity is usually related to the momentum diffusity,  $K_m$  as:  $K_H = K_m / \phi$  where  $\phi$  is a stability correction (e.g., Mehlenbacher and Whitfield, 1977). Thom (1975) discusses various models regarding  $K_m$  and u(z) and suggested that virtually any reasonable assumptions regarding  $K_m$ could produce a realistic profile of wind speed. Hence, for this work, the model proposed independently by Thom (1971) and Landsberg and James (1971) in which  $K_m$  is assumed constant throughout the canopy will be used. Thus  $K_m = ku_{\downarrow}(h - d)$  where k is the Van Karman constant (0.40);  $u_{\star}$  is the friction velocity at the top of the canopy; h is the tree height; and d is the zero plane displacement. This choice for  $K_m$  is consistent with the following wind profile (Thom, 1971; Landsberg and James, 1971): u(z) = $u_h(1 + a_d(1 - \xi))^{-2}$  where  $u_h$  is the wind speed at the top of the tree,  $a_d$  is a constant; and  $\xi$  is the normalized height (z/h). Hence, thermal diffusivity is treated also as a constant. The stability correction  $\phi$  should be near unity during and shortly after rainfall events. Thus, the average heat flux inside the canopy is

$$-\frac{\overline{K_{H}}}{h}\frac{d\psi}{d\xi} = -\frac{1}{h(1-\xi_{0})} \int_{\xi_{0}}^{1} (K_{H}\frac{d\psi}{d\xi})d\xi = -\frac{K_{H}}{h}\left(\frac{\psi(1)-\psi(\xi_{0})}{1-\xi_{0}}\right)$$
(17)

10

\*

where  $\xi_0$  is the value of  $\xi$  at the bottom of the canopy, which in this case is not the ground level.

Thus, because  $K_H$  is a constant it is possible to express the average heat flux in terms of the boundary conditions  $\psi(1)$  and  $\psi(\xi_0)$  instead of resorting to the more difficult approach of solving (16) either numerically or analytically. Finally, the average evaporation rate throughout the canopy is assumed to be directly proportional to the average sensible heat flux. Therefore, substituting for  $K_H$  and  $\psi$  in (17), the average evaporation rate from a wet forest canopy can be written as

$$\overline{E} = b_1 b_2 u_h \left[ \theta(1) - \theta(\xi_0) \right] \tag{18}$$

where  $\mathbf{b}_1 = - [\mathbf{b}(h-d)\mathbf{k}^2]/\{\phi h \ln [(h-d)/z_0]\}$  and  $\mathbf{b}_2 = \rho c_p/[\lambda(1-\xi_0)]$ ; where  $z_0$  is the roughness length; b is the proportionality constant between the latent and sensible heat fluxes and is related to the Bowen ratio; and  $\lambda$  is the latent heat of vaporization. To express  $\overline{E}$  in terms of  $u_H$  instead of  $u_*$  it is assumed that  $u_h = (u_*/\mathbf{k}) \ln[(h-d)/z_0]$ . This last relationship is an oversimplification because the logarithmic profile is usually valid only at heights much greater than the tree height; but for the purpose of this study it is assumed to be valid at tree height.

To employ (18),  $b_1$  is assumed to remain relatively constant and that variability in evaporation rates from one storm to the next is due to differences in temperatures and wind speeds.

The storms of 5 and 9 December, 1981 were used to estimate  $b_1$ , by comparing the computed  $E_0$  (Table I) with expression 18. For the December 5 storm  $u_h = 0.23 \,\mathrm{m \, s^{-1}}$  and the potential temperature difference was 0.27 K; for the December 9 storm  $u_h = 0.21 \,\mathrm{m \, s^{-1}}$  and the difference in potential temperature was 0.47 K. Thus, for these two storms  $b_1$  was estimated to be 0.28 and 0.19, respectively. This close agreement is encouraging, but it may also be coincidental because the evaporation rates,  $E_0$ , are very nearly identical for each storm. Only further testing and comparison with other methods will show whether this approach is useful or not. Furthermore, this method should be tested on other stands and other tree species since the parameter  $b_1$  may be influenced by stand and species' characteristics. A tower would also be useful in testing this new method of estimating evaporation rates, because with a tower direct estimates of  $b_1$  could be made.

This completes the discussion on evaporation. The purpose of the section was to give a brief outline of a new approach for estimating evaporation rates from forest canopies and some preliminary results.

#### CONCLUSIONS

The new drip expression presented here explicitly includes the rain rate and does give a significant improvement over Rutter-like drip expressions. However, it has only been tested in old-growth Douglas fir forests. If the model can be extended to other forests, it should give further credibility to the notion that the impact of falling rain drops upon foliage surfaces, which dislodges previously intercepted rain drops, plays a major part in the drip of intercepted rain water from forest canopies.

The analytical model derived from the new dynamic model is considerably less sensitive to the exact numerical values of the drip parameters, requires less detailed information than the dynamic model, and gives good agreement with observed gross interception loss for a single old-growth Douglas fir tree. The analytical model, without empirical constants or a computer program, predicted the gross interception loss for the 20 storms tested to be 4% more than the observed gross interception loss. The Rutter-like drip-model, eq. 6, also gave acceptable results for the gross interception loss. However, this latter model required a computer program and its two empirical constants had to be determined on a yearly basis rather than at the level of individual storms. Hence the model parameters may be valid only for the data set used here. The analytical model assumes that the storm is sufficient to saturate the tree and, roughly speaking, that the evaporation rate is less than the interception rate in the forest canopy. At this site, these conditions were almost always fulfilled with each rainstorm. The adaptation of the analytical model to other forests should be much easier than with the dynamic model.

Although the evaporation model proposed here also requires further testing, the initial results are encouraging. This evaporation model was derived assuming that the thermal diffusivity is a constant throughout the tree canopy and that the Bowen ratio and other micrometeorological parameters can be suitably averaged over the duration of a rainstorm. This new approach to estimating evaporation rates from wet canopies may be applicable in forests with deep canopies, where the temperature difference between the top and the bottom of the canopy is fairly large.

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#### APPENDIX

This appendix provides 3 mathematical proofs: Proof I gives eq. 10 which was used to evaluate  $d_0$  for each individual storm tested; Proof II establishes that the parameter  $\beta$  is bounded i.e.,  $\frac{1}{2} \leq \beta \leq 1$ ; and Proof III shows that eq. 15 predicts that I increases as  $R_0$  increases i.e.,  $\partial I/\partial R_0 > 0$ .

# Proof I

The present model of water storage on foliage surfaces is given as follows

$$dS/dt = (1 - p) R(t) - [D_0 + d_0 R(t)] S/S_c - E_0 \min(S/S_c, 1)$$
(A1)

The evaporation rate used here is the same as that given by (7) except that it has been restated in a more compact form. Integrating (A1) between the time the storm begins t = 0 and the time the drip ceases  $t = T_1$  gives the following equation

$$S(T_{1}) - S(O) = (1 - p)P_{G} - D_{0} \int_{0}^{1} (S/S_{c})dt - d_{0}$$

$$\int_{0}^{T_{1}} R(t)S/S_{c}dt - S_{evap}$$
(A2)

Assuming that the canopy is dry when the rain begins and that the storm completely saturates the canopy, then:  $S(T_1) = S_c$ ; S(O) = 0; and

 $S_{\text{evap}} = P_G - P_N - S_c$ 

Substituting these last three equalities into (A2) and solving (A2) for  $d_0$  gives

$$d_{0} = (P_{N} - pP_{G} - D_{0}T_{1}\overline{S}/S_{c})/\{\int_{0}^{1} [R(t)S/S_{c}]dt\}$$
(A3)

Where  $\overline{S} = 1/T_1 \int_0^{T_1} Sdt$  and is the average amount of water stored on the canopy surfaces during the time  $T_1$ .

# Proof II

From the main text  $\beta = [A + (1 - A) \ln (1 - A)]/A^2$ , where  $A = (D_0 + d_0R_0 + E_0)/[(1 - p)R_0]$  and 0 < A < 1. Because A is bounded,  $\ln (1 - A)$  can be expanded in a Taylor's series. By collecting and cancelling terms where appropriate, it follows that  $\beta = 1/2 + \sum_{i=1}^{\infty} A^i/[(i + 1) (i + 2)]$ . This expression for  $\beta$  is monotonically increasing so that  $\beta$  is bounded below by  $\beta = 1/2$ . Likewise  $\beta$  is bounded above by  $\lim_{A \to 1} \beta$ . Using the original expression to evaluate the latter limit gives:  $\lim_{A \to 1} \beta = 1$ . This result immediately follows since  $(1 - A) \ln (1 - A)$  approaches 0 as A approaches unity.

#### Proof III

From the text eq. 15 gives  $I = S_c(1 - \beta E_0/[(1-p)R_0]) + E_0T_1$ . Differentiating this expression for I with respect to  $R_0$  gives:  $\partial I/\partial R_0 = \{S_c E_0/[(1-p)R_0]\}\{\beta/R_0 - \partial\beta/\partial R_0\}$ . Thus, it is sufficient to show that  $\partial\beta/\partial R_0 < 0$  for all values of  $R_0$  in order to establish that  $\partial I/\partial R_0 > 0$ , which can be done by employing the following identity:  $\partial\beta/\partial R_0 = (d\beta/dA)$  $(\partial A/\partial R_0)$ . Using the Taylor series expression for  $\beta$  given in the last proof it follows that  $d\beta/dA = \sum_{i=1}^{\infty} iA^{i-1}/[(i+1)(i+2)] > 0$  for all values of A since A > 0. Using the definition of A gives  $\partial A/\partial R_0 = -[d_0/(1-p)][(1-p)A/d_0 - 1]$ . However,  $A = d_0/(1-p) + (D_0 + E_0)/[(1-p)R_0] > d_0/(1-p)$ . Therefore,  $A > d_0/(1-p)$  which is equivalent to  $[(1-p)A/d_0 - 1] > 0$ . Hence,  $\partial A/\partial R_0 < 0$  for all values of  $R_0$  and so  $\partial \beta/\partial R_0 < 0$  for all values of  $R_0$ .

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