WATER RESOURCES RESEARCH, VOL. 16, NO. 1, PP. 210–216, FEBRUARY 1980

Water Storage on Forest Foliage: A General Model

WILLIAM J. MASSMAN

Biology Department, University of Oregon, Eugene, Oregon 97405

A model for the prediction of water accumulation on forest foliage is presented. It is general enough to include linear, exponential, or logarithmic forms for the buildup of accumulated water and to go from any one of these three limiting forms to any other in a continuous manner by varying a single dimensionless parameter. This equation unifies into a single form many other models used to predict water accumulation (from either rain or fog) on forest foliage. An exploration of the underlying assumptions relating drip rate to interception intensity shows how these other models differ from one another. Evaporation is included in the model by using another dimensionless parameter which may be very useful in regions where only an approximate estimate of the evaporation rate is available. However, to include evaporation correctly into the model it is necessary to define the water storage capacity first in order to properly interpret the model predictions.

INTRODUCTION

There have been several different approaches recently to the problem of water accumulation from both rain and fog on tree foliage. For instance, *Rutter et al.* [1972, 1975] and *Rutter and Morton* [1977] used a dynamical model to predict the amount of water intercepted by trees during rain, whereas a more empirical approach was taken by *Jackson* [1975] with rain and *Merriam* [1973] with fog when they used logarithmic or exponential regressions to fit observed data. The purpose of this note is to present a general equation which is based on simple dynamical principles and which displays linear, logarithmic, or exponential behavior. This equation unifies the different approaches taken by the above mentioned authors and by the proper choice of a single parameter reduces to forms they used in their works. It is hoped that this discussion will help illuminate both the physical basis of the storage process and the nature of the differences in the approaches that these other researchers have taken. After the dynamical basis of the model is fully explored, evaporation is included. Because the model is quite flexible, exact details of the evaporation rate are not critical. Hence the model may be very useful in regions where the evaporation rate is only approximately known.

DYNAMICAL BASIS

For the purposes of this section, evaporation will not be discussed, hence the rate of water stored within a given foliage volume can be written as follows:

$$dS/dt = I(t) - D(t) \tag{1}$$

where S is the amount of stored water, $I(t)$ is the rate of foliage interception (henceforth called the interception intensity), and $D(t)$ is the rate at which water is leaving that volume after being intercepted (henceforth called the drip rate, which does not include stem flow); t is time.

The basic assumption is that the drip rate is proportional to the amount of water stored in the tree such that when the tree is initially dry, the drip rate is zero, and when maximum storage is reached, the drip rate is equal to the interception intensity. Besides this dependence upon the amount of water stored in the foliage, the drip rate can be either explicitly or implicitly proportional to the interception intensity. Explicit proportionality means that the drip rate is directly proportional to the interception intensity, i.e., $D(t) \propto I(t)$. Implicit proportionality means that the drip rate is not directly proportional to the interception intensity but is influenced by it only so far as the interception intensity affects the amount of intercepted water. This section assumes the former, and a subsequent section will explore the consequences of the later assumption.

One such form for explicit proportionality is given by the linear relationship

$$D(t) = I(t)(S/S_c) \tag{2}$$

where S_c is the maximum storage. However, a more general relationship is given as follows:

$$D(t) = I(t)\left(\frac{e^{\alpha(S/S_c)} - 1}{e^{\alpha} - 1}\right) \tag{3}$$

Here α is a dimensionless constant which depends on tree species and meteorological conditions and will be discussed in greater detail later. Note that when the foliage is dry the drip rate is zero and when the tree is saturated the drip rate is equal to the interception intensity, so that the boundary conditions are met. Also note that as α approaches zero, (3) will approach the form given by (2). Now assuming that the interception intensity is constant (denoted by I_0) and that the tree is initially dry when the rain begins, the solution to (1) using the form given by (3) is

$$S/S_c = -1/\alpha \ln [(1 - e^{-\alpha}) \cdot \exp \{-\alpha I_0 t/[S_c(1 - e^{-\alpha})]\} + e^{-\alpha}] \tag{4}$$

The solution, $S = S(t)$, given by (4) is dependent upon the parameter α in a manner which is illustrated in Figure 1. In this figure, both axes have been normalized. On the vertical axis, S is expressed in fractions of S_c, and on the horizontal axis, time is expressed as $(I_0/S_c)t$. The axes have been normalized for greater generality and to remove the need to discuss particular events or particular canopies, so that the emphasis can be on the basic assumptions and dynamics. Note that the independent variable is actually $(I_0/S_c)t$ and not exactly time. The true elapsed time required for reaching some given level of saturation is proportional to S_c/I_0, which is determined by the interception intensity and the nature of the foliage. In this graph the dashed upper most curve is a limiting form of the solution and corresponds to the situation in which the rate of storage is equal to the interception intensity, and once the foliage is sat-

Paper number 9W1280.
0043-1397/80/009W-1280$01.00

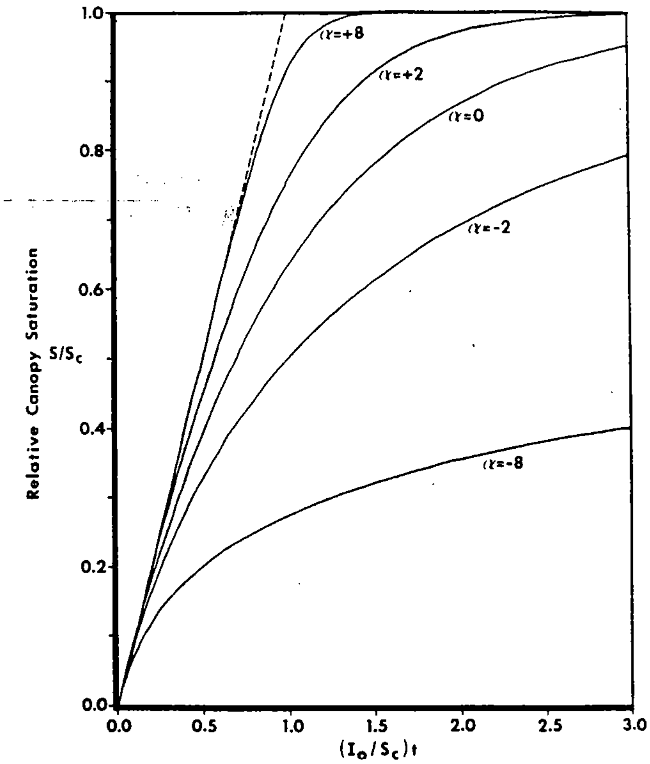

Fig. 1. Water accumulation on foliage without evaporation as predicted by (4) for various values of α. The stored water, S, is shown as a function of time and expressed as a fraction of the water storage capacity S_c. The time axis is dimensionless and is expressed in units of $(I_0/S_c)t$. The dashed line corresponds to the most rapid accumulation of water possible on the foliage.

urated, the excess drips out. This limiting form of (4) corresponds to a 'cup' which does not allow water to drain from it until it is filled, then it overflows. All other solutions are various types of 'leaky cups' which allow water to flow out as it accumulates in the cup.

Limiting Forms

To gain further insight into the nature of (4), several special limits will be examined. During an initial phase immediately after the rain begins, all curves on Figure 1 are nearly linear. As time progresses, all solutions eventually approach unity, but the parameter α strongly influences how these curves approach this limit. For a large positive value of α ($\alpha \to +\infty$), the solution becomes linear; for a large negative value of α ($\alpha \to -\infty$), the solution becomes logarithmic; and for a small value of α ($\alpha \to 0$), the solution is exponential:

$$S/S_c \to (I_0/S_c)t \qquad (\alpha \to +\infty) \tag{5a}$$

$$S/S_c \to \frac{1}{|\alpha|}\ln|\alpha| + \frac{1}{|\alpha|}\ln(I_0/S_c)t \qquad (\alpha \to -\infty) \tag{5b}$$

$$S/S_c \to 1 - e^{-(I_0/S_c)t} \qquad (\alpha \to 0) \tag{5c}$$

Here $|\alpha|$ is the absolute value of α. Equation (5b) is valid only for a large but finite value of $|\alpha|$; otherwise S/S_c vanishes for finite time. Generally, the limiting forms (5a) and (5b) are approached very closely whenever $|\alpha| > 4$.

Equation (5a) simply means that as α becomes large, the solution will approach the cup solution. Equation (5b) is the leakiest cup solution and is a logarithmic form. Here I_0t is the total intercepted rainfall in the period t. Since the interception intensity is usually just some fraction of the rainfall intensity, R_0, (5b) can be written in the form $a + b \ln t + c \ln R_0$, which is the form chosen by *Jackson* [1975] for rainfall interception. He found that as the rainfall intensity increased, i.e., at larger values of R_0, the interception increased when the duration remained fixed. He offered several reasonable explanations for this dilemma, but this result is less surprising when interpreted in light of (5b), which says that the interception S must increase as R_0 increases. Hence his result may be an artifact of the form of the equation he chose to fit the data. Since the logarithmic form dynamically corresponds to the leakiest cup solution, it may be more appropriate for hydrophobic foliage surfaces.

Equation (5c) is the case $\alpha = 0$, and this form of the solution is used extensively in both rain and fog interception studies [e.g., *Merriam*, 1973]. In that same paper, Merriam states, 'The exponential decay generally underpredicted the early buildup (of stored water) on the pine.' Considering (4), this means that positive α predicts the accumulation more accurately than α equal to zero. Figure 1 shows that the accumulation rate is greater for α positive than for α equals zero.

Assuming that our approach is realistic, the following interpretation of α can be made. Once the storage capacity S_c is correctly estimated, then all other natural or meteorological variability in the accumulation of water on plant foliage must be related to α. It is probably true that α is affected by temperature, wind speed, rain intensity, and foliage characteristics as well as precipitation type such as fog or rain. The parameter α may also be a function of time as events with a rainstorm evolve with time, but an approach based on some free parameter may eventually improve our ability to predict water accumulation on plant foliage.

Discussion of Implicit Proportionality

The assumption made earlier was that the drip rate is explicitly proportional to the interception intensity. This section will assume that the drip rate is implicitly proportional to it. This means that the drip rate is influenced by the interception intensity only so far as the latter affects the amount of interception, S/S_c. With this assumption instead of (3), the following expression is used:

$$D(t) = D_0\left(\frac{e^{\alpha(S/S_c)} - 1}{e^{\alpha} - 1}\right) \tag{6}$$

where D_0 is a constant, termed the drainage constant, which, like α, may depend on foliage characteristics and meteorological conditions. Expressing the drip rate $D(t)$ in the form of (6) rather than (3) is the mathematical expression for the implicit proportionality assumption. For a constant interception intensity, I_0, and the form given in (6), if $D_0 \neq I_0$, then some redefinition of S_c may be necessary because S_c may no longer be a maximum value for the tree storage capacity.

Figures 2a and 2b show how the drainage constant D_0 can influence the interpretation of S_c. These figures give the solution to (1) using (6) for various choices of α and the ratio of the drainage constant to the interception intensity, D_0/I_0. When this ratio is one, the solution is determined by α only and reduces to that shown in Figure 1. These figures have been normalized in the same manner as Figure 1.

Figures 2a and 2b show that the solution is similar to that shown in Figure 1 except that the asymptotic value of S/S_c is no longer unity but something that depends on the parameter α as well as the drainage constant D_0. The values of D_0/I_0 are

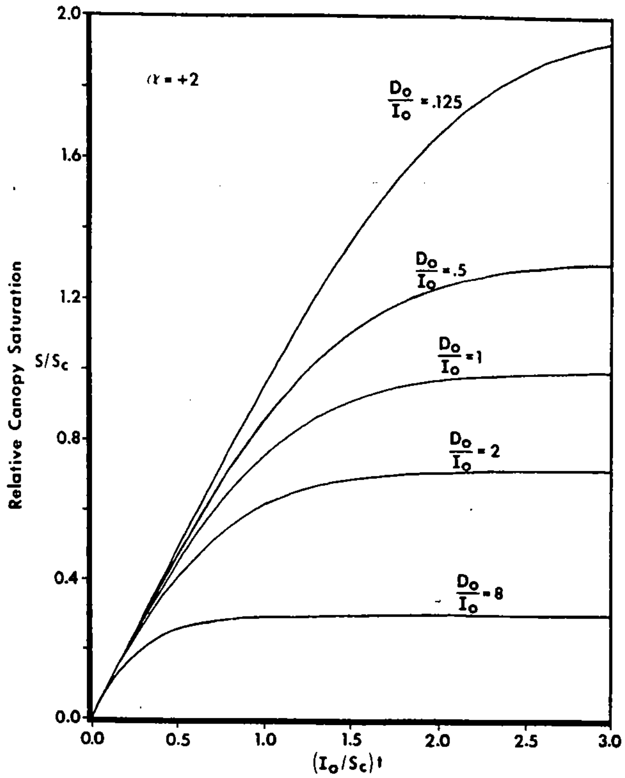

Fig. 2*a*. Water accumulation without evaporation on foliage as predicted using the implicit proportionality assumption, equation (6), with $\alpha = +2$ for various values of the ratio D_0/I_0. Both axes are the same as given in Figure 1.

different on each figure because not all combinations of α and D_0/I_0 give acceptable solutions. A more detailed mathematical discussion of this solution and its asymptotic values is given in the appendix.

From these figures it is clear that the choice of D_0 will strongly affect interpretation of S_c. For example, if the ratio D_0/I_0 is small and α is greater than two, then S/S_c tends asymptotically to something greater than one, which is consistent with the interpretation that the water holding capacity S_c is the minimum amount of water needed, when there is no evaporation, to cover the tree foliage with a thin stable film of water. Any water in excess of this amount will eventually drip out. On the other hand, in the previous discussion the storage capacity S_c was defined to be a maximum value.

The interpretation of the water holding capacity S_c as a minimum and the particular choice for α and the ratio D_0/I_0 is of special interest as they are related to the model used by *Rutter et al.* [1972, 1975] and *Rutter and Morton* [1977]. The parameter α used in this work is numerically equal to their drainage parameter b. (In this particular case their drainage coefficient b is identical to α/S_c, and their value of S_c was about 1 mm.) Their choice of D_0 also insures D_0/I_0 is less than one. This is because they chose D_0 so small that measurements of drip rates less than it are unreliable. Finally, their value of b which is based on observation is sufficiently large to ensure that e^b is much greater than one. They also noted some variability of their drainage parameter b from one storm to the next, which is not surprising if the physical interpretation of α offered earlier is correct.

After the water holding capacity has been reached and with the proper identification of parameters, then (6) reduces to the following expression:

$$D(t) = D_0 e^{-\alpha(1-S/S_c)} \tag{7}$$

which is identical to the Rutter drip rate. However, at times when the tree is dry, the Rutter model, equation (7), predicts a small amount of drip. This criticism has been pointed out by *Calder* [1978]. On the other hand, the model proposed in (6) does not have this difficulty; the drip is zero when the tree is dry. Furthermore, because the Rutter model violates the boundary condition, the drainage constant D_0 must be chosen small to insure that the drip is negligible when the tree is dry. This is not a problem with the model proposed here, since D_0 can take on large or small values.

To summarize this section, the shape of the solution shown in Figure 2 are similar to those presented in Figure 1 except that they tend asymptotically to a value different from unity which is primarily caused by the implicit proportionality assumption. It was also shown that the Rutter drip equation is an approximation to the more general one presented in (6) and that unlike the Rutter equation, (6) avoids a finite drip when the tree is dry.

EVAPORATION

To include evaporation the continuity equation given by (1) can be written as follows:

$$dS/dt = I(t) - D(t) - E(t) \tag{8}$$

where $E(t)$ is the evaporation rate. Generally, the evaporation rate is affected not only by meteorological conditions but by the amount of intercepted water present on the foliage surfaces. The Rutter model makes the evaporation rate proportional to the relative canopy saturation, S/S_c, whenever S/S_c is less than or equal to one, and otherwise the evaporation rate is not influenced by the amount of intercepted water. *Calder*

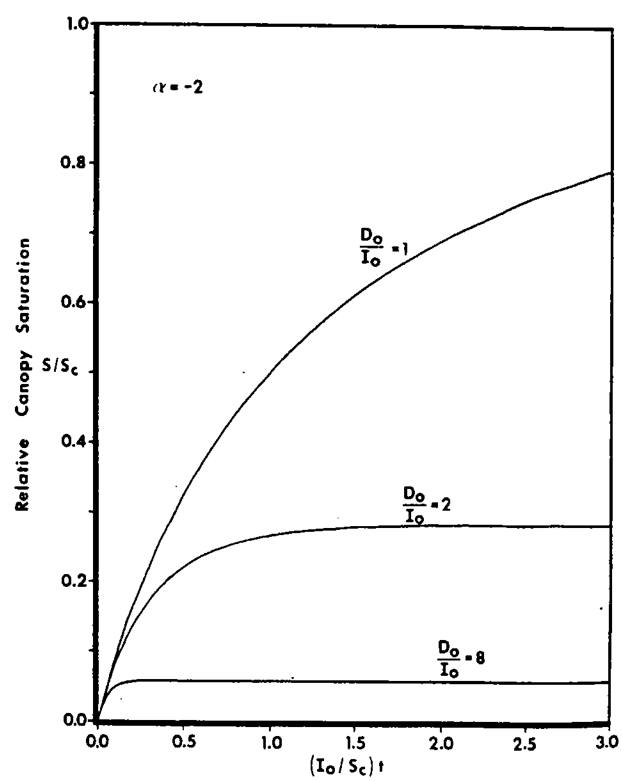

Fig. 2*b*. Same as Figure 2*a* except for $\alpha = -2$. Not all values of D_0/I_0 as in the previous figure are shown because not all solutions are acceptable. This is discussed in more detail in the appendix.

[1978] does not explicitly assume that the evaporation rate is proportional to S/S_c, but he does allow parameters of his model to vary depending on whether S/S_c is greater than one or not. However, both these models do employ evaporation rates as estimated by the Penman-Monteith equation. For the model described in this section, detailed evaporation rates such as the Penman-Monteith equation provides are not needed, and so it may be useful in areas where complete instrumentation excludes the use of the Penman-Monteith equation or when advection of energy may strongly influence evaporation rates.

The evaporation rate $E(t)$ is written as follows:

$$E(t) = E_0\left(\frac{e^{\beta(S/S_c)}-1}{e^{\beta}-1}\right) \tag{9}$$

where E_0 and β are constants which depend on tree species and foliage characteristics as well as the meteorological conditions. Where detailed information is available on the evaporation rate, it is possible to substitute the Penman-Monteith equation for E_0. However, this paper will assume E_0 is constant.

Note than when β is zero, (9) reduces to the form similar to that used in the Rutter model when S/S_c is less than one.

Two simple examples of how (9) can be used will now be given. The first is when the tree is completely wet but unsaturated, i.e., when the rain and drip have stopped but evaporation continues, and the second is when the tree is initially dry and it begins to rain.

For the first example, (8) can be written as follows:

$$dS/dt = -E_0\left(\frac{e^{\beta(S/S_c)}-1}{e^{\beta}-1}\right) \tag{10}$$

The solution to this equation is given in Figure 3 for various choices of β. The axes on this figure have again been normalized with the time scale given by $(E_0/S_c)t$, and the dashed straight line is the limiting solution which corresponds to the constant evaporation of accumulated water without regard to the amount of water present. In this example the definition for the water storage capacity must be that minimum amount necessary to completely wet the tree. Because the parameter β now offers greater freedom in simulating the evaporation, detailed information on the evaporation rate is no longer necessary. For example, *Hancock and Crowther* [1979] measured the evaporation of water from a wet but unsaturated canopy using branches as cantilever weighing machines. To fit their data they essentially used β equal zero and a different evaporation rate, E_0, during different parts of the day. So they in a sense used two curves to fit their data; but using (10), it may be possible to fit their data with only one curve and two parameters, β and E_0. This in effect would mask the natural variability of the evaporation rate. Hence detailed information on the evaporation rate is no longer as important.

In the cases where the ratio S/S_c is greater than one, the Rutter model suggests that the evaporation rate is not influenced by the amount of water stored in the tree. These cases occur when the tree is dripping or when it is raining. It would be interesting to see how accurately the evaporation model suggested by (9) can be extended to include the situations where the amount of water stored in the tree exceeds the minimum amount required to wet all the surfaces. Since (9) does not need detailed information on the evaporation rate, one curve may fit data where the ratio S/S_c exceeds one for portions of the data and is less than one for other portions.

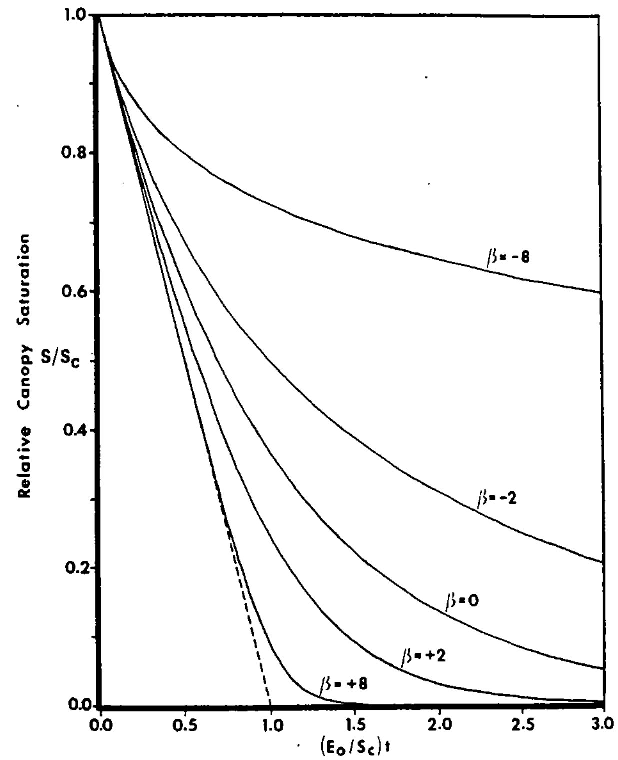

Fig. 3. Evaporation of accumulated water predicted by (10) for a completely wet tree which is no longer dripping or intercepting water for various choices of β. The time axis is dimensionless and expressed in units of $(E_0/S_c)t$. The dashed line corresponds to the most rapid evaporation of water possible from the foliage.

The second example of how (9) might be used is that of a tree which is initially dry when it begins to rain. This is essentially the same situation used in previous sections except with evaporation included. Thus the general way to write (8) is as follows:

$$dS/dt = I_0 - D_0\left(\frac{e^{\alpha(S/S_c)}-1}{e^{\alpha}-1}\right) - E_0\left(\frac{e^{\beta(S/S_c)}-1}{e^{\beta}-1}\right) \tag{11a}$$

Where the interception intensity is assumed to be constant.

When the tree is fully wet, i.e., when the water in the tree, S, is greater than or equal to the storage capacity S_c, then (11*a*) is replaced by (11*b*):

$$dS/dt = I_0 - D_0\left(\frac{e^{\alpha(S/S_c)}-1}{e^{\alpha}-1}\right) - E_0 \tag{11b}$$

However, as discussed earlier, it is possible that this extra condition need not be imposed, since the parameter β offers a great deal of flexibility; but (11*b*) will be used in this paper where appropriate, since it does seem to be more realistic. In applications where the evaporation rate is low relative to the interception rate or where it can only be approximately estimated then using (11*a*) for all values of the storage ratio S/S_c, whether greater or less than one, may be easier as there would only be one equation to deal with rather than two.

For simplicity, β is assumed to be the same as α, although this may not be the case generally, since β should depend upon how the intercepted water is exposed to the environment and α depends on how the intercepting surfaces are oriented relative to the direction of rainfall or fog movement. For the case where E_0 is low compared to the interception intensity, the parameter β has little influence, and so it can easily be taken to be the same as α without much effect. Generally, without the simplification of α equal β, the solution to (8) is similar to that shown in Figure 1 except that (*a*) the asymptotic values of S/S_c are always less than they are without evaporation, and they decrease as β decreases; (*b*) it takes longer to saturate the foliage than when evaporation is not included, and the saturation time lengthens as β increases; and (*c*) the drip rate always approaches a value less than the interception intensity. Assuming $\alpha = \beta$, then (11*a*) can be written as follows:

$$dS/dt = I_0 - (D_0 + E_0)\left(\frac{e^{\alpha(S/S_c)} - 1}{e^{\alpha} - 1}\right) \qquad (12)$$

Equation (11*b*) remains unaltered, although as discussed earlier, it may be easier and just as accurate to extend the evaporation model, equation (9), into regions where the storage ratio S/S_c is greater than one. However, it is not clear how this would impact the assumption that α and β are equal. Figures 4*a* and 4*b* show the solution to the model (11*b*) and (12) for various choices of the ratio E_0/I_0. For Figure 4*a* the drip ratio D_0/I_0 is 0.125 and $\alpha = +2$ and for Figure 4*b* the drip ratio D_0/I_0 is the same but $\alpha = -2$. For these figures the results are similar to those shown in Figures 2*a* and 2*b* which is not surprising, since (12) is mathematically identical to the model discussed in the previous section on implicit proportionality; except that D_0 is replaced by $(D_0 + E_0)$. Furthermore, replacing D_0 by $(D_0 + E_0)$ is independent of whether the drip rate is implicitly or explicitly proportional to the interception intensity. On the other hand, this expression is not completely identical to the model discussed in the previous section. The definition of the water holding capacity, S_c, in some way influences whether the implicit or explicit assumption is made

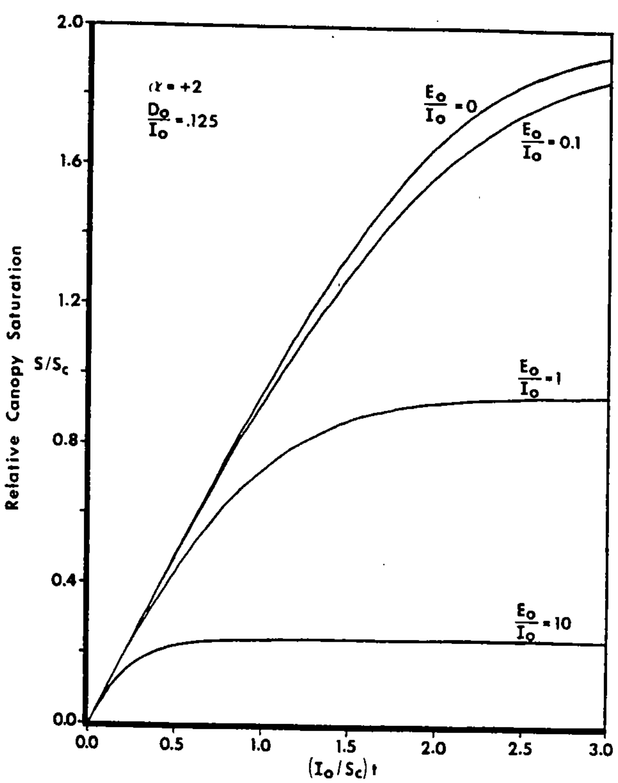

Fig. 4*a*. Water accumulation on foliage including the influence of evaporation as predicted by (12) and (11*b*), using $\alpha = +2$ and $D_0/I_0 = 0.125$ and for various choices of E_0/I_0. The axes are the same as in Figure 1.

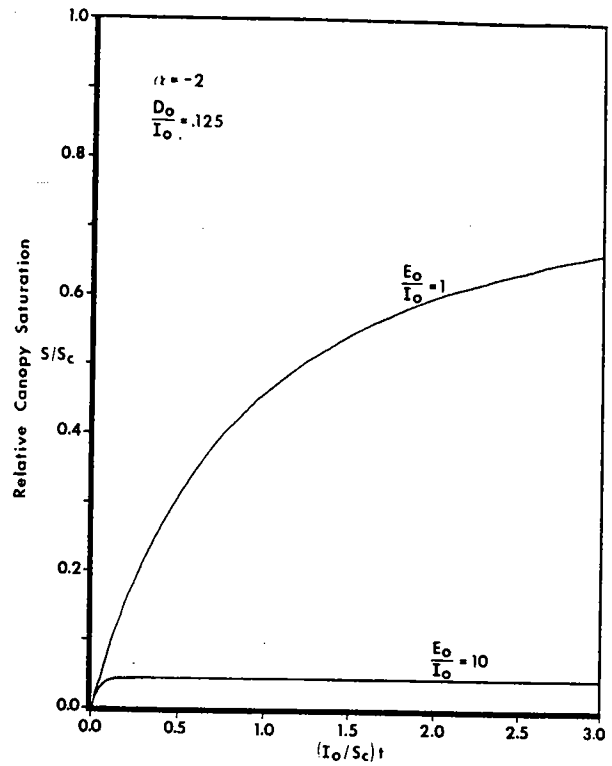

Fig. 4*b*. Same as Figure 4*a* except for $\alpha = -2$. Not all values of D_0/I_0 as in the previous figure are shown because not all solutions are acceptable. This is discussed in greater detail in the appendix.

and vice versa. Once that question is resolved, evaporation can only reduce the amount of water stored in the tree compared to when evaporation is not taking place, and the tree will take longer to become saturated with evaporation than without it. For this example though, the mathematical similarities end when the amount of stored water is greater than the storage capacity and (11*b*) is used, which assumes that once the tree is completely wet and S/S_c equals one, then the evaporation rate is not influenced by the amount of stored water. More details on the nature of the solution to (12) are given in the appendix, where the implicit proportionality assumption is further discussed.

The greatest value of the model proposed by (12) may be that detailed information on the evaporation rate is now not so important. It is certainly possible to include as much detailed information as is available, but it may not be necessary because the other model parameters D_0 and α provide a great deal of flexibility. Equation (12) is probably most accurate when the evaporation rate is small compared to the interception intensity; but only comparison to observations will tell for sure. For cases of high evaporation rates and low interception intensities or advection of energy, (11*a*) may offer a better approach.

DISCUSSION

It is important to consider the question of implicit or explicit proportionality before using the model suggested here. Making the drip rate explicitly proportional to the interception intensity appears to be valid when the foliage is dry initially or when the foliage is partially wet but not dripping, like those situations investigated by *Merriam* [1973] and *Jackson* [1975]. On the other hand, this assumption cannot remain valid always because the drip can continue well after the rain has stopped. When this happens, the implicit proportionality assumption is more useful and hence may apply to those cases where the tree may not dry out completely before the rain begins again or in regions where the rainfall intensity is highly variable and it is not always possible beforehand to know whether the tree is completely dry or not. It is also important to have some understanding of how the storage capacity S_c is to be interpreted, since the definition of S_c can strongly influence whether the implicit or explicit proportionality assumption is made. For example, if S_c is to be the minimum amount required to wet the tree, then only the implicit proportionality assumption can be made in order to insure that the amount of water in the tree, S, can be greater than S_c. However, without data and detailed studies of drip rates at different interception intensities and low evaporation rates, it may not be possible to decide which approach is correct. Ultimately, the appropriate choice may depend upon the specific situation under investigation or the simplicity and ease that one approach has over the other when simulating or predicting the amount of water in tree canopies.

Once questions are made concerning the interpretation of the water holding capacity S_c and which proportionality assumption is appropriate, evaporation can easily be included in a way that makes detailed information on the evaporation rate unnecessary. This approach could be of great benefit in regions where only a crude estimate of the evaporation rate is available. However, it is important that these other basic questions be resolved first in order to correctly interpret the model predictions.

Constant interception intensity and evaporation rates were assumed mainly for simplicity so attention could be focused on the approach and its underlying dynamics and on its strengths and weaknesses. For application where the interception intensity and evaporation rates are variable, a simulation would easily include this extra variation as the Rutter model has demonstrated. These extra effects would not fundamentally alter the discussions and conclusions drawn assuming a constant interception intensity and constant evaporation rates.

SUMMARY

A general equation for the prediction of water accumulation from either rain or fog on forest foliage was presented which unified equations used by *Rutter et al.* [1972, 1975] and *Rutter and Morton* [1977] and the more empirical ones used by *Merriam* [1973] and *Jackson* [1975]. This model is based on a single dimensionless parameter and under the proper limits and assumptions predicts linear, logarithmic or exponential accumulation of water on plant foliage. An exploration of the underlying assumptions relating drip rate to interception intensity showed that the Rutter model is based on a slightly different assumption of how the drip rate is related to interception intensity than are the more empirical models of *Jackson* [1975] and *Merriam* [1973]. Evaporation is then included in the model in a manner that makes detailed information on the evaporation rate unnecessary. This is done by using another dimensionless parameter and hence may be of value in regions where lack of complete instrumentation may preclude the use of the Penman-Monteith equation for estimating evaporation of water from plant foliage surfaces. However, before including evaporation into the model, it is necessary to define the water storage capacity S_c and to choose the proportionality assumption for the drip rate which is consistent with that definition.

APPENDIX

The purpose of this appendix is to briefly discuss the mathematical nature of the solution to the model assuming the implicit proportionality assumption. With this assumption the conservation equation can be written as follows:

$$dS/dt = I_0 - D_0\left[\frac{e^{\alpha(S/S_c)} - 1}{e^{\alpha} - 1}\right] \quad (13)$$

and the general solution to this equation is given as follows:

$$S/S_c = -1/\alpha \ln [g^{-1}(1 - e^{-\alpha}) \exp \{-g\alpha I_0 t/S_c(1 - e^{-\alpha})\} + e^{-\alpha}(g^{-1}D_0/I_0)] \quad (14)$$

where $g = 1 - e^{-\alpha} + D_0/I_0 e^{-\alpha}$ and is written into (14) for convenience. Note that when D_0 equals I_0, g reduces to one, and (14) reduces to (4). Since g, which is determined by α and D_0/I_0, now appears in the exponent, its sign influences the nature of the solution. In particular, those values of α and D_0/I_0 which cause the solution, S/S_c, to increase to infinitely large values are to be excluded. These excluded values are given by the following inequalities:

$$\alpha < 0 \qquad 0 < (D_0/I_0) \leq 1 - e^{-|\alpha|} \quad (15)$$

Otherwise all other values of α and D_0/I_0 are acceptable. Thus all positive values of α are acceptable, and only a portion of negative values of α are unacceptable.

The solution (14) now has a much different asymptotic value than (4), and it depends upon D_0 as well as α. This asymptotic value is given as follows:

$$S/S_c \rightarrow 1 - \frac{1}{\alpha}\ln\frac{D_0}{I_0} + \frac{1}{\alpha}\ln\left(1 + \frac{D_0}{I_0}e^{-\alpha} - e^{-\alpha}\right) \quad (16)$$

which is valid for all acceptable values of α and D_0/I_0. Note that the argument $(1 + (D_0/I_0)\, e^{-\alpha} - e^{-\alpha})$ must be positive for all acceptable solutions, which follows from the conditions discussed earlier.

The asymptote is of interest primarily as it illuminates something of the nature of the model. In particular, when D_0/I_0 is less than one, then this model will predict that the amount of water intercepted by the tree will be greater for larger values of interception intensity, and vice versa, when D_0/I_0 is greater than one, this model will predict that the amount of water intercepted by the tree will decrease with larger values of interception intensity. This is an artifact of the model only. When D_0/I_0 is equal to one, then S/S_c will approach unity.

Therefore it is important to have an appropriate understanding of the water storage capacity before deciding whether the implicit or explicit assumption is to be made. If S_c is to be a maximum value, then the explicit proportionality assumption can be made or at least D_0 chosen greater than I_0. On the other hand, if S_c is to be a minimum amount required to wet the tree surface, then the implicit proportionality assumption must be made with the condition that D_0/I_0 be less than one, so that the intercepted water, S, can eventually exceed the storage capacity S_c.

With the inclusion of evaporation for a relatively simple case it was shown that D_0 could be replaced by $(D_0 + E_0)$. Hence in a mathematical sense the above discussion of the model is valid with evaporation also. However, it is important beforehand to have a definition of the storage capacity S_c with its attendant conditions on the drainage constant D_0, so that evaporation will be independent of the definition of the storage capacity and vice versa.

In fact the inclusion of evaporation will make the conditions on α and D_0 given in (15) less stringent, since the sum of D_0 and E_0 now replaces D_0 in that inequality. However, the mathematical similarities end if the evaporation rate is no longer influenced by the amount of stored water in the tree. This is most likely to happen when the storage capacity S_c is exceeded, although, again, there may be application for which it is simpler not to apply this condition.

Acknowledgments. The author wishes to thank Phil Sollins for his comments and discussions in the formulation of this paper and Bill Hopper and Terry Montlick who assisted in computer programing. This research was supported under grant DEB78-03583 from the National Science Foundation.

REFERENCES

Calder, I. R., Transpiration observations from a spruce forest and comparisons with predictions from an evaporation model, *J. Hydrol.*, *38*, 33–47, 1978.

Hancock, N. H., and J. M. Crowther, A technique for the direct measurement of water storage on a forest canopy, *J. Hydrol.*, *41*, 105–122, 1979.

Jackson, I. J., Relationships between rainfall parameters and interception by tropical forests, *J. Hydrol.*, *24*, 215–238, 1975.

Merriam, R. A., Fog drip from artificial leaves in a fog wind tunnel, *Water Resour. Res.*, *9*, 1591–1598, 1973.

Rutter, A. J., and A. J. Morton, A predictive model of rainfall interception in forests, III, Sensitivity of the model to stand parameters and meteorological variables, *J. Appl. Ecol.*, *14*, 567–588, 1977.

Rutter, A. J., K. A. Kershaw, P. C. Robins, and A. J. Morton, A predictive model of rainfall interception in forests, I, Derivation of the model from observations in a plantation of corsican pine, *Agr. Meteorol.*, *9*, 367–384, 1972.

Rutter, A. J., A. J. Morton, and P. C. Robins, A predictive model of rainfall interception in forests, II, Generalization of the model and comparison with observations in some coniferous and hardwood stands, *J. Appl. Ecol.*, *12*, 367–380, 1975.

(Received April 25, 1979;
revised August 22, 1979;
accepted August 28, 1979.)