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Comment on "Critical flow constrains flow hydraulics in mobile-bed streams: A new hypothesis" by G. E. Grant

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1. Introduction

This paper discusses the challenging ideas proposed by *Grant* [1997] on steep movable bed channel flows. First the definition of critical flow are revisited. Then the calculation of bed shear stress in near-critical flows are discussed. New experimental data are presented, and they highlight the three-dimensional variations of boundary shear stress. Further, it is believed that the applicability of Grant's hypothesis is restricted because of the confusion between critical and near-critical flows.

The discusser wishes to congratulate *Grant* [1997] for some interesting and challenging ideas on steep movable bed channel flows. This paper comments on the definition of critical flow (based on *Grant* [1997, equation 1], the calculation of bed shear stress in near-critical flows, and the applicability of the Grant's hypothesis. I hope that the information will assist to refine the Grant's developments.

2. Definition of Critical Flow Conditions

Historically critical flow conditions were defined as the flow properties at the singularity of the backwater equation [*Belanger*, 1828; *Bazin*, 1865]. In the general case of a channel of nonconstant shape and longitudinal bed slope, the one-dimensional form of the energy equation (i.e., the backwater equation) becomes

$$\frac{\partial H}{\partial x} = \frac{\partial d}{\partial x} \cos \theta - d \sin \theta \frac{\partial \theta}{\partial x} + \frac{\partial z_o}{\partial x} - \alpha \frac{Q^2}{gA^3} \frac{\partial A}{\partial x} = -S_f \tag{1}$$

where H is the mean total head (i.e., average over the flow cross-section area), x is the longitudinal coordinate in the flow direction, d is the flow depth measured normal to the channel bottom, θ is the longitudinal bed slope, z_o is the bed elevation, α is the kinetic energy correction coefficient (i.e., Coriolis coefficient), Q is the water discharge, A is the cross-section area, B is the free-surface width, S_f is the friction slope,

$$S_f = f \frac{1}{D_H} \frac{V^2}{2g} \tag{2}$$

f is the Darcy friction factor, D_H is the hydraulic diameter, and V is the mean flow velocity ($V = Q/A$). For a constant channel shape, $\partial A = B \partial d$ and (1) becomes

$$\frac{\partial d}{\partial x} = \frac{\sin \theta - S_f + d \sin \theta \dfrac{\partial \theta}{\partial x}}{\cos \theta - \alpha \dfrac{Q^2 B}{gA^3}} \tag{3}$$

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More recently [*Bakhmeteff*, 1912, 1932; *Henderson*, 1966], critical flow conditions were defined as the flow properties (depth and velocity) for which the mean specific energy is minimum for a given flow rate and channel cross-section shape. The mean specific energy is defined as

$$E = d \cos \theta + \alpha \frac{V^2}{2g} \tag{4}$$

assuming a hydrostatic pressure distribution. E is similar to the energy per unit mass, measured with the channel bottom as the datum. In the general case of a nonrectangular channel [e.g., *Henderson*, 1966, p. 51], the mean specific energy is minimum for

$$\frac{\partial E}{\partial d} = \cos \theta - \alpha \frac{Q^2 B}{gA^3} = 0 \tag{5}$$

Today this second definition of critical flow (i.e., equation (3)) is commonly used. Equations (5) and (3) for θ constant yield the criterion for critical flow conditions

$$\alpha \frac{Q^2 B}{g \cos \theta A^3} = 1 \tag{6}$$

Equation (6) is more general than *Grant*'s [1997] equation (1) which did not take into account the bed slope effect nor the cross-section shape. Here, α is larger than unity although rarely exceeds 1.15, and the ratio $\alpha/\cos \theta$ typically ranges between 1 and 1.3 in natural streams.

3. Near-Critical Flows and Bed Shear Stress

Near-critical flows may be defined as flow situations characterized by the occurrence of critical or nearly critical flow conditions over a "reasonably long" distance and time period. The specific energy/flow depth diagram shows that near the critical flow conditions, a very small change of energy (e.g., caused by a bottom or sidewall irregularity) can induce a very large change of flow depth. Near-critical flows are indeed characterized by the development of large free-surface undulations [e.g., *Imai and Nakagawa*, 1992; *Chanson and Montes*, 1995; *Chanson*, 1995, 1996; *Montes and Chanson*, 1998]. Experimental observations showed further that "undular flows" may take place for $0.3 \leq Fr \leq 3$, but this range of flow conditions could be broader depending upon the boundary conditions.

Here I performed new experiments in a 20-m-long fixed-bed channel of rectangular cross section ($W = 0.25$ m) to investigate the boundary shear stress under an undular hydraulic jump. The bed shear stress was measured with a Prandtl-Pitot tube ($\varnothing = 3.3$ mm) used as a Preston tube (see appendix).

Bed shear stress measurements under an undular jump (in a fixed-bed channel) are presented in Figure 1, in which the bed shear stress at various positions across the channel ($z/W = 0$ at the sidewall, $z/W = 0.5$ on the centerline) is plotted as a

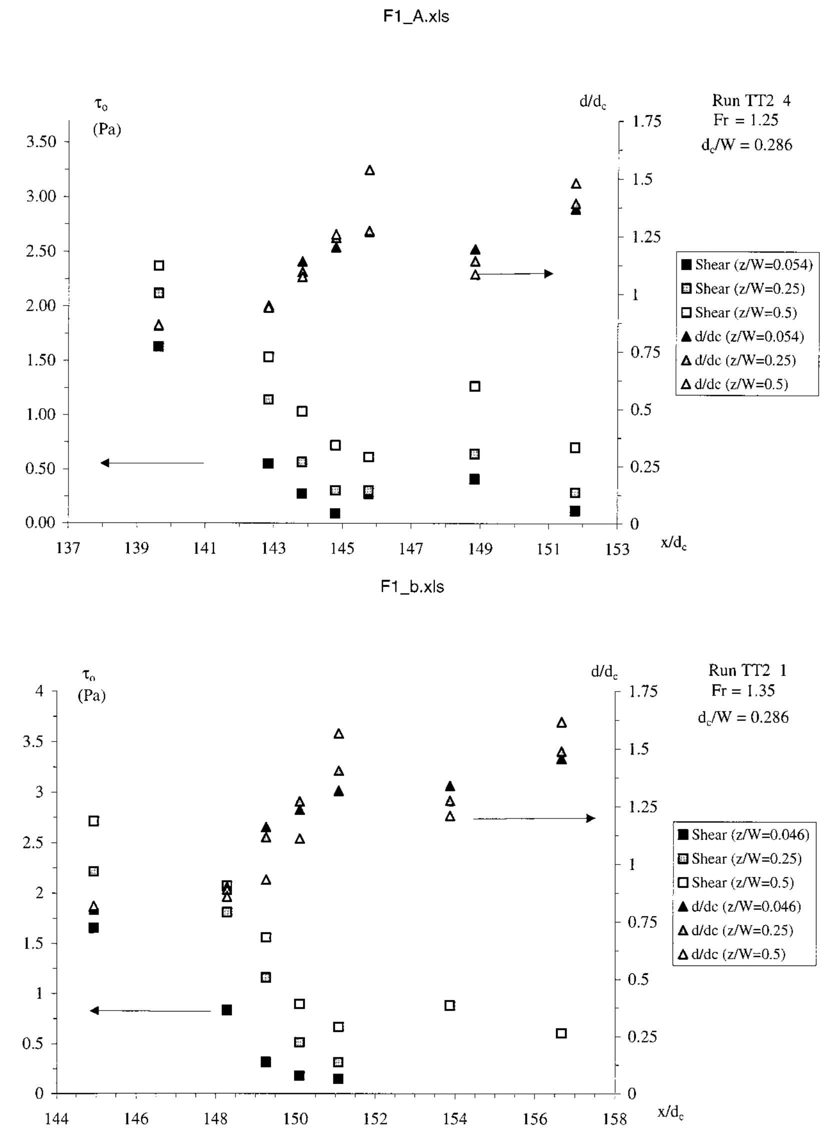

Figure 1. Bed shear stress along an undular hydraulic jump. (a) Flow conditions: $Fr = 1.25$, $d_c/W = 0.286$, $W = 0.25$ m (Run TT2_4). (b) Flow conditions: $Fr = 1.35$, $d_c/W = 0.286$, $W = 0.25$ m (Run TT2_1).

function of the dimensionless distance x/d_c, where x is the distance from the channel intake and d_c is the critical flow depth. On the same graph, the dimensionless flow depth d/d_c is shown also.

The discussers' data show large fluctuations of bed shear stress in the crosswise and longitudinal directions. Typically, the bed shear is minimum below the wave crests and maximum underneath the wave troughs. The results are similar to the findings of *Imai and Nakagawa* [1992].

The results have direct implications to natural mobile-bed channels. Considering a flat movable-bed stream, an undular flow might take place during a flood event or in an estuary during a period of the tide. Below the free-surface undulations, the movable bed becomes subjected to an nonuniform boundary shear stress distribution (Figure 1), and, as a result, erosion may take place underneath the wave troughs while accretion occurs below the wave crests. Altogether the conditions are favorable for the development of standing-wave bed forms (in phase with the free-surface standing waves).

Note further that the boundary shear stress is consistently smaller near the wall (black squares in Figure 1) than on the channel centerline (white squares in Figure 1). Hence sediment motion is likely to be more intense near the channel centerline than next to the banks.

4. Discussion

Grant's [1997] hypothesis and the corollary that "assumption of critical flow would essentially replace the standard flow resistance equations" might not be an ultimate achievement. Even if the limiting state of mobile-bed flows is critical flow, can the flow conditions be predicted accurately?

The discusser does not think so. Indeed the present discussion has highlighted some aspects of the complexity of undular flows. Figure 1 highlights that the boundary shear stress distribution along undular flow (in a fixed-bed channel) is affected by large changes in both the lateral and longitudinal directions. In Figure 1 the bed shear stresses fluctuate by 1 order of magnitude in the longitudinal and lateral directions. *Grant*'s [1997, equations (3), (8), and (13)] use of empirical flow resistance correlations imply a "uniformly distributed" boundary shear stress. Such an assumption is refuted by this paper's results (Figure 1).

Further, *Grant*'s [1997] developments deriving from his "hypothesis" are based on empirical correlations which do not reflect the physical nature of the energy losses nor the interactions between the flow and the movable boundaries. In an alluvial channel, the boundary friction is related to the skin friction (or grain-related friction) and to the form losses caused by the bed forms. Although the skin friction shear stress (or effective shear stress) may be calculated accurately, the estimate of the bed-form shear stress is difficult. Fundamental experiments in irregular open channels [*Kazemipour and Apelt*, 1983] showed that the form losses could account for up to 92% of the total loss. For undular flows, there is little experimental data to predict accurately the form losses associated with the standing wave bed forms, but this situation does not justify the use of "overall bed shear stress" correlations.

In summary, *Grant*'s [1997] hypothesis of critical flow cannot be considered as an ultimate simplification. The present discussion and the works of *Imai and Nakagawa* [1992], *Chanson* [1995], *Chanson and Montes* [1995], and *Montes and Chanson* [1998] highlighted the complexity of undular flows.

Appendix: Calibration of Pitot Tube

The Pitot tube was calibrated in uniform equilibrium flows. The calibration curve (Pitot tube velocity versus boundary shear stress) was best fitted by

$$\tau_o = 3.1821 V_b^{2.0275} \tag{7}$$

was similar to calibration curves obtained by *Preston* [1954] and *Patel* [1965] and *McIntosh* [1990] who used the same experimental channel. Here τ_o is the boundary shear stress and V_b is the velocity measured by the Pitot tube lying on the bed.

Notation

A cross-section area (m^2).
B free-surface width (m).
D_H hydraulic diameter (m).
d flow depth (m).
d_c critical flow depth (m).
E mean specific energy (m).
f Darcy friction factor.
g gravity constant (m/s^2).
H mean total head (m).
Q discharge (m^3/s).
S_f friction slope.
V mean flow velocity (m/s), $V = Q/A$.
V_b velocity (m/s) measured close to the channel bed.
W channel width (m).
x longitudinal coordinate (m).
z cross-wave coordinate (m), $z = 0$ at the wall.
z_o bed elevation (m).
θ mean channel slope.
τ_o bed shear stress (Pa).

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