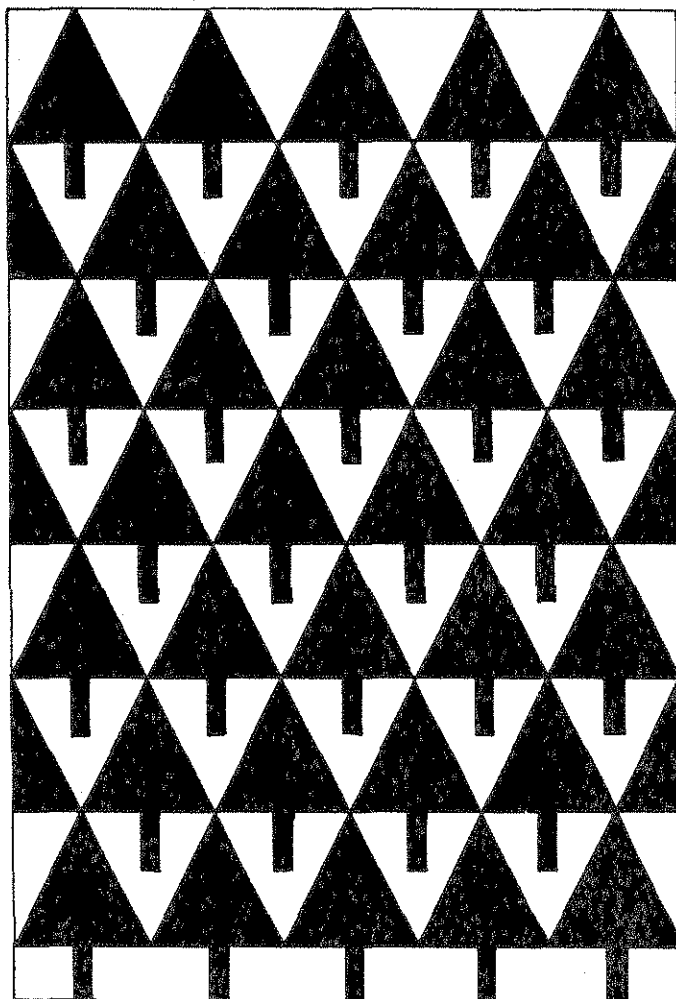


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SITE INDEX AND HEIGHT GROWTH CURVES

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FOR MANAGED,
EVEN-AGED STANDS OF DOUGLAS-FIR
EAST OF THE CASCADES
IN OREGON AND WASHINGTON



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Site Index and Height Growth Curves for Douglas-fir in Managed, Even-Aged Stands East of the Cascades in Oregon and Washington

Reference Abstract

Cochran, P. H.

1979. Site index and height growth curves for managed, even-aged stands of Douglas-fir east of the Cascades in Oregon and Washington. USDA For. Serv. Res. Pap. PNW-251, 16 p., illus. Pacific Northwest Forest and Range Experiment Station, Portland, Oregon.

Height growth and site index curves and equations for managed, even-aged stands of Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) east of the Cascade Range in Oregon and Washington are presented. Data were collected in stands where height growth apparently has not been suppressed by high density or top damage.

KEYWORDS: Increment (height), site index, stem analysis, even-aged stands, Douglas-fir, *Pseudotsuga menziesii*, Oregon (eastern), Washington (eastern).

RESEARCH SUMMARY

Research Paper PNW-251

1979

Height growth and site index curves and equations for managed, even-aged stands of Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) east of the Cascade Range in Oregon and Washington were derived from stem analysis data from 22 sample plots in Oregon and 10 sample plots in Washington.

Height growth curves give estimates of expected heights at different ages for stands of

known site index. Site index curves give estimates of site index of managed stands where only present breast height age and present total height are available.

The appropriate curves provide valid estimates of site index and potential height growth of the tallest trees for stands where height growth has not been retarded by high density or

related factors. They *do not* represent the average of existing stands.¹ The height growth curves are most appropriate for use in constructing yield tables for managed, even-aged stands of Douglas-fir or mixed conifers where Douglas-fir is a significant component.

These results stem from a study undertaken as part of the

Douglas-fir Tussock Moth Expanded Research and Development Program. The purpose of the study was to determine potential production of stands susceptible to attack by tussock moth.

Curves are based on measurements of the tallest tree for its breast high age in a 1/5-acre plot.

¹Summerfield, Edward R. Site index and height growth of Douglas-fir and ponderosa pine in eastern Washington. Washington State Department of Natural Resources Report 38 (in preparation for publication).

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Introduction

Over 9 million of the 20 million acres of commercial forest land in eastern Oregon and eastern Washington is in mixed conifer forests.² Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) is a primary component of many of these forests occurring either in pure stands or mixed primarily with white fir (*Abies concolor* (Gord. & Glend.) Lindl.), grand fir (*Abies grandis* (Dougl.) Lindl.), western larch (*Larix occidentalis* Nutt.), or ponderosa pine (*Pinus ponderosa* Laws.). The site index and height growth curves presented here are for Douglas-fir in pure or mixed, even-aged, managed stands where relatively low density, lack of top damage, and absence of vegetative competition early in the life of the stand permit full height development.

A managed stand is being manipulated toward some goal, usually a "target" average diameter and height within a set time frame. To attain these goals, the manager will often use some combination of precommercial and commercial thinnings and perhaps early suppression of competing vegetation.

No stands exist that have been under such management throughout a rotation. Therefore, I deliberately chose stands for sampling that approximated densities believed desirable in managed stands, not merely the average of existing stands. Height growth from 4.5 feet upward was apparently never reduced by stand density, vegetative competition, or top damage in these stands.³

² 1 acre = 0.4047 hectare.

³ 1 foot = 0.3048 meter.

The height growth curves and site index curves presented here are two separate sets of curves constructed for two different objectives. Height growth curves are used in construction of yield tables for describing top height as a function of age and site index. Site index curves are used to determine an index to potential production from current height and age (Curtis et al. 1974).

Methods

Data Collection

Data came from thirty-two 1/5-acre circular plots (fig. 1) with these characteristics:

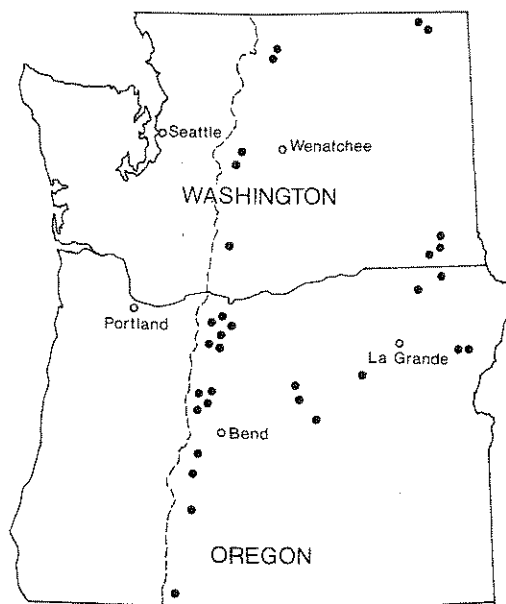


Figure 1.--Distribution of plots used in construction of curves for Douglas-fir.

1. The average breast high age was greater than 50 years. At ground line, the ages of the youngest trees were at least 80 percent of the ages of the oldest trees.

2. The crown canopy was closed or nearly closed at the time of sampling. Stumps were absent; and mortality, if evident, was due to suppression and was less than 5 percent of the plot volume. Volume growth patterns for the plot determined from stem analysis of at least 12 sample trees across the range of diameter classes indicated that the highest periodic annual volume increment had occurred within the last 5 to 15 years. These factors indicate that there was no severe competition between trees in the past.

3. The dominant trees on the plot did not contain a group of narrow annual rings, which would indicate stress in the past.

4. Trees were not infected with disease, and no visible signs of insect defoliation were present.

5. Dominant trees did not exhibit crook in the bole, and internodal lengths did not indicate past top damage. Some plots were rejected after sampling because abrupt breaks in the height growth curves suggested substantial top damage 30 or more years earlier, even though this damage was not apparent at the time of selection.

6. Clumps of trees were not sampled. The plots were in homogeneous stands, and each plot had a buffer strip equivalent in width to tree height.

Diameter at breast height (d.b.h.) was measured for each tree on each plot, and 12 to 15 trees of each species on each plot were felled to determine past periodic annual increments for the plot. Included in this group were the three largest diameter trees. If taller trees existed on the plot, they were felled and their stem analysis data used

for construction of the site index and height growth curves but not in the volume determinations. Past volume and basal area growth for nonsample trees on the plot were predicted from their basal areas. Deliberately using the tallest tree in forming the predictive equations would have produced a biased overestimate of volume growth.

The three to five tallest Douglas-fir trees at the time of sampling were sectioned at a 1-foot stump, at 4.5 feet (bh), 10 feet, and then at 10-foot intervals up the stem after total height was measured. Sections at ground line were also taken from at least two of the largest diameter trees, two of the smallest diameter trees, and one tree with close to the mean diameter. Rings were counted for all sections and recorded for the appropriate height.

Curve Construction

An age of 50 years at breast height (4.5 feet) was chosen as the index age. For each plot, heights of the three to five tallest trees were plotted as a function of bh age⁴ for each tree on a single sheet of graph paper. The bh age for each tree was used as the independent variable in the initial plotting rather than average bh age because height growth from 0 to 4.5 feet seems to be greatly influenced by competition immediately adjacent to a seedling and perhaps by early animal damage. Thus, it is possible for dominant trees to

⁴ Because, on the average, internodes were cut in half, these height-over-age curves contain compensating errors but overall are not biased because age at 4.5 feet determined from ring counts was the independent variable.

be the same age at ground line but to differ as much as 10 years at 4.5 feet. Use of an average bh age for plotting heights results in an underestimate of the height growth potential of the site under management. Shifts in the tree of maximum height for its bh age among the sample trees occurred on 50 percent of the plots. Freehand curves were drawn for each tree and the highest points at each decadal age interval were used in subsequent construction of curves. Site index for each plot was defined as the tallest height at bh age 50. This procedure resembles that of Dahms (1963), except that Dahms used an average age for the plot as the independent variable in plotting the course of height growth for the tallest trees.

From this point, the methodology outlined by Barrett (1978) is used; it includes the recent improvements in curve construction methods suggested by Curtis et al. (1974) and Dahms (1975). A brief outline is presented in the appendix.

For the 10 sample plots in Washington, site indexes ranged from 55 to 105.4. The 22 sample plots in Oregon had site indexes ranging from 52.7 to 106.4. The average site index was 84.47:

<u>Number of plots</u>	<u>Site Index</u>
4	50-59
4	60-69
3	70-79
3	80-89
13	90-99
5	100-110

Results

Detailed results are given in the appendix along with an out-

line of the methods of curve construction. Some understanding of curve construction leads to an appreciation of how they should be used so the appendix is recommended reading even for the occasional user.

Estimating Site Index

Site index curves are used to indicate the potential productivity of land. The curves here give the estimated height of the tallest tree when the breast high age of that tree is 50 years. Many of the same plot qualifications used in this study are applicable in selecting plots for measuring site index. The following steps and procedures are recommended for estimating the site index of a managed stand.

1. Select suitable plots with the following characteristics:

(a) Even-aged at the ground line (practically, there are no older remnants from earlier stands and the present stand is one storied).

(b) No visible signs of disease or insect attack to reduce height growth.

(c) No narrow ring groups to indicate suppression.

(d) Consistent internodal lengths on taller trees.

(e) No remnant understory vegetation or suppression mortality to indicate competition early in the life of a stand.

2. Establish boundaries of a 1/5-acre plot with a prespecified shape.

3. Measure the height of the three to five tallest trees on the plot.

4. Extract increment cores from these trees to determine their breast-high age.

5. Using the breast-high age and total height for each tree, determine a site index value for each tree by one or more of the following alternatives.

(a) Use figure 2 for rough field estimates.

(b) Obtain a more precise estimate by using the appropriate a and b values in table 1 to solve the equation,

$$\text{Site index} - 4.5 \text{ feet} = a + b (\text{height} - 4.5 \text{ feet}). \quad (1)$$

(c) For those wishing to program the estimating procedure in a calculator, the appropriate equation in the appendix can be used.

6. Record as the site index value for the plot, the highest of the three to five values determined.

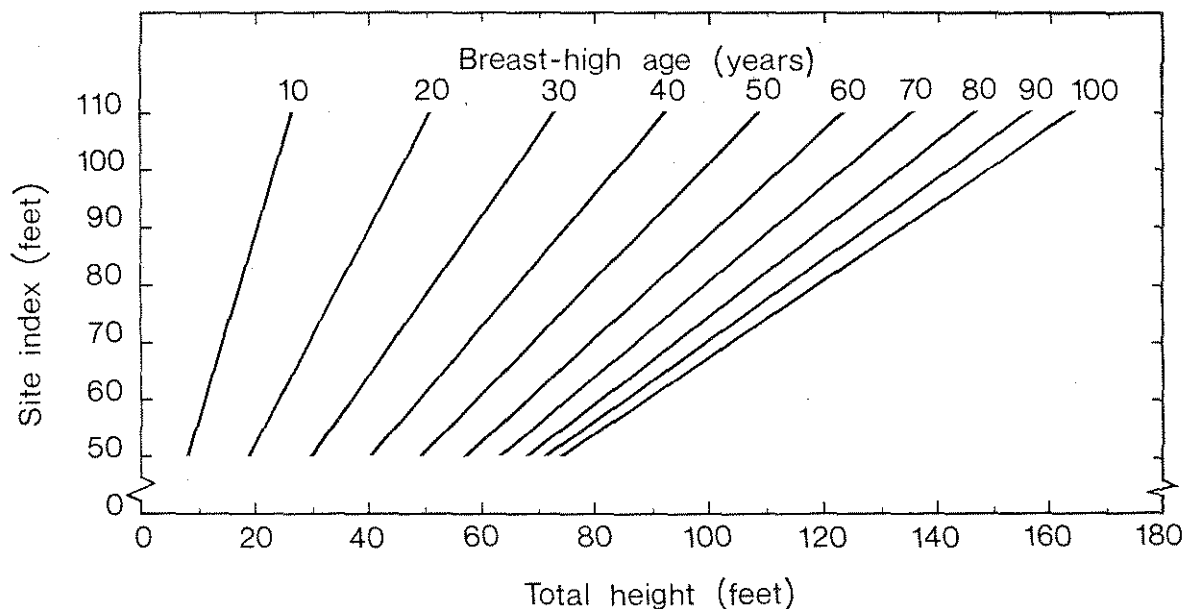


Figure 2.--Site index curves for manage, even-aged stands of Douglas-fir east of the Cascades in the Pacific Northwest.

Table 1--Values for *a* and *b* by years for the family of regressions^{1/} for estimating site index for Douglas-fir east of the Cascades in the Pacific Northwest

Breast-high age	Years between decades																			
	0		1		2		3		4		5		6		7		8		9	
	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b
Years																				
10	32.217	3.235	30.319	2.936	38.679	2.778	27.069	2.602	25.550	2.451	24.113	2.319	22.751	2.204	21.457	2.103	20.228	2.012	19.056	1.931
20	17.940	1.858	16.875	1.792	15.585	1.731	14.886	1.676	13.958	1.625	13.069	1.579	12.219	1.535	11.406	1.495	10.627	1.458	9.881	1.423
30	9.168	1.390	8.484	1.359	7.830	1.331	7.204	1.304	6.605	1.278	6.032	1.254	5.483	1.231	4.959	1.209	4.458	1.188	3.979	1.168
40	3.522	1.150	3.085	1.132	2.760	1.114	2.274	1.098	1.896	1.082	1.537	1.067	1.196	1.053	.872	1.039	.565	1.025	.274	1.012
50	0	1	-.260	.988	-.505	.976	-.735	.965	-.951	.954	-1.153	.944	-1.342	.934	-1.518	.924	-1.681	.914	-1.832	.905
60	-1.971	.896	-2.098	.887	-2.214	.879	-2.319	.870	-2.413	.862	-2.496	.854	-2.569	.847	-2.632	.839	-2.685	.832	-2.729	.825
70	-2.764	.818	-2.789	.811	-2.806	.804	-2.814	.798	-2.814	.791	-2.805	.785	-2.789	.779	-2.764	.773	-2.732	.767	-2.693	.761
80	-2.646	.756	-2.593	.750	-2.532	.745	-2.465	.740	-2.391	.734	-2.311	.729	-2.224	.724	-2.132	.719	-2.033	.714	-1.928	.709
90	-1.818	.705	-1.703	.700	-1.582	.695	-1.455	.691	-1.324	.687	-1.187	.682	-1.046	-.961	-.900	.674	-.749	.669	-.594	.665
100	-.434	.661																		

^{1/}To estimate site index, measure height of the 3 tallest trees per 1/5-acre plot. Determine breast-high age for each of these trees. Select appropriate *a* and *b* values above. Substitute values in the equation, Site index - 4.5 feet = *a* + *b* (height - 4.5 feet). For example, for a tree 53 years old at breast height and 60 feet in total height, solve the equation, SI - 4.5 = -0.735 + 0.965 (60 - 4.5), for a site index of 57.3. Determine the site index for each of the 3 trees. The highest site index determined is the site index for the 1/5-acre plot.

Estimating the Course of Height Growth for Stands of a Given Site Quality

Height growth curves define the average pattern of height development for the tallest trees in stands of a given site quality. They are appropriately used for construction of yield tables but do not provide optimum estimates of site index from measured height and age in an existing stand (Curtis et al. 1974).

Three alternatives exist for estimating the anticipated height of the tallest trees of a stand on land of known site index:

1. Rough field estimates through use of figure 3.
2. A more precise estimate using a_1 and b_1 values in table 2 in the equation,
Height - 4.5 feet = $a_1 + b_1$ (site index - 4.5 feet).
3. An estimating procedure programed in a calculator using the equation shown in the appendix.

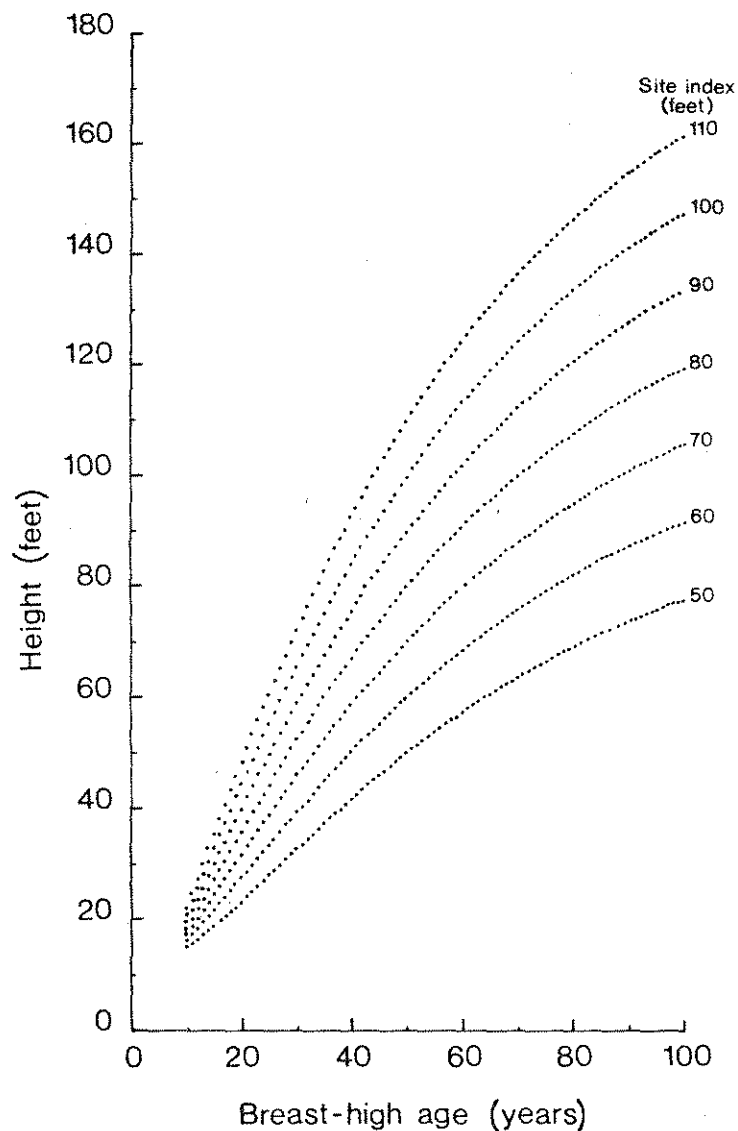


Figure 3.--Height growth curves for the tallest trees in managed, even-aged stands of Douglas-fir east of the Cascades in the Pacific Northwest.

Table 2--Values for *a* and *b* by years for the family of regressions^{1/} for estimating height of the tallest trees in a newly established stand of Douglas-fir east of the Cascades where site index and age are known

Breast-high age	Years between decades																			
	0		1		2		3		4		5		6		7		8		9	
	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b
Years																				
10	5.573	0.115	4.407	0.149	3.924	0.182	3.216	0.214	2.575	0.245	1.997	0.276	1.475	0.306	1.006	0.335	0.586	0.364	0.209	0.392
20	-.125	.419	-.422	.446	-.683	.472	-.911	.497	-1.108	.522	-1.127	.546	-1.418	.570	-1.534	.593	-1.627	.615	-1.698	.637
30	-1.749	.659	-1.781	.680	-1.795	.700	-1.192	.720	-1.773	.740	-1.740	.759	-1.692	.778	-1.632	.796	-1.560	.814	-1.477	.832
40	-1.383	.849	-1.279	.866	-1.166	.882	-1.045	.898	-.915	.913	-.778	.929	-.634	.944	-.484	.958	-.328	.972	-1.166	.986
50	0	1.000	.172	1.013	.348	1.026	.578	1.039	.711	1.051	.898	1.064	1.087	1.075	1.279	1.087	1.474	1.098	1.670	1.110
60	1.869	1.120	2.069	1.131	2.270	1.141	2.473	1.151	2.676	1.161	2.881	1.171	3.085	1.181	3.290	1.190	3.495	1.199	3.701	1.208
70	3.906	1.216	4.110	1.225	4.315	1.233	4.518	1.241	4.721	1.249	4.923	1.257	5.124	1.264	5.324	1.272	5.523	1.279	5.720	1.286
80	5.916	1.293	6.110	1.300	6.303	1.306	6.494	1.313	6.684	1.319	6.871	1.325	7.057	1.331	7.241	1.337	7.422	1.343	7.601	1.349
90	7.779	1.354	7.954	1.359	8.126	1.365	8.297	1.370	8.465	1.375	8.630	1.380	8.793	1.385	8.954	1.389	9.112	1.394	9.267	1.398
100	9.420	1.403																		

^{1/}Height at a future date of the tallest portion of a young stand may be estimated on land of known site index by selecting *a* and *b* values for the appropriate breast-high age. Substitute *a*₁ and *b*₁ values in the equation, Height - 4.5 feet = *a*₁ + *b*₁ (site index - 4.5 feet), for the particular breast-high age wanted. For example, for the height of the tallest tree in the stand at breast-high age 85 on land with a known site index of 100, solve the equation, HT - 4.5 = 6.871 + 1.325 (100 - 4.5), for a total height of 137.9 feet.

Application

In this study, site index is a number representing the height of the tallest tree for its breast-high age of 50 years on a 1/5-acre plot. Since site has been found to be closely correlated with volume (Spurr 1952), site index (as discussed here) will later be used in a yield study to categorize volume productivity potentials of managed stands of Douglas-fir east of the Cascades. Height objectively reflects site where undamaged stands are not overstocked. Stands managed for maximum production of usable wood, in contrast to natural stands, probably will not be overstocked to the point of substantially reducing height growth. Therefore, use of these curves should be restricted to even-aged stands where height growth competition between trees has been held to a minimum.

Typical examples of when the curves should not be used are:

1. Precommercially thinned stands showing a tight core of rings.
2. Commercially thinned stands with numerous stumps indicating a high initial density.
3. Plantations with large numbers of trees, thinned long after severe competition between trees occurred.
4. Stands that have been subjected to Douglas-fir tussock moth, spruce budworm, or other insect attacks that resulted in loss of top growth.

Only a limited amount of land is now under intensive management and fits the constraints of these curves. Greatest use will probably be in forecasting future stand performance in simulation

models. Stand projections should assign appropriate lesser heights to other than the tallest trees in the stand. This problem of height assignment is currently being investigated.

A partial judgment of the reliability of the curves can be made by the r^2 values and the standard errors of the estimates shown in the appendix. Equation fit can be judged from figures 8 and 9 in the appendix.

Literature Cited

- Barrett, James W.
1978. Height growth and site index curves for managed, even-aged stands of ponderosa pine in the Pacific Northwest. USDA For. Serv. Res. Pap. PNW-232, 14 p., illus. Pac. Northwest For. and Range Exp. Stn., Portland, Oreg.
- Curtis, Robert O., Donald J. Demars, and Francis R. Herman.
1974. Which dependent variable in site index-height-age regressions? For. Sci. 20(1):74-87.
- Dahms, Walter G.
1963. Correction for a possible bias in developing site index curves from sectioned tree data. J. For. 61(1):25-27.
- Dahms, Walter G.
1975. Gross yield of central Oregon lodgepole pine. In Management of lodgepole pine ecosystems symposium proceedings, p. 208-232, illus. David M. Baumgartner, ed. Wash. State Univ. Coop. Ext. Serv., Pullman.
- Snedecor, George W., and William G. Cochran.
1967. Statistical methods. 6th ed. 593 p., illus. Iowa State Univ. Press, Ames.
- Spurr, Stephen H.
1952. Forest Inventory. 476 p. Ronald Press Co., New York.

Appendix

For both site index and height growth curves, a curve of average height for the samples as a function of age at 4.5 feet is constructed. This height curve is then adjusted to the desired site index, using the linear relationship existing between height and site index at any age, with appropriate estimates of slope and intercept. The curves are different because slope and intercept values of the equations

$$SI - 4.5 \text{ feet} = a + b (HT - 4.5 \text{ feet}) \text{ and}$$

$$HT - 4.5 \text{ feet} = a_1 + b_1 (SI - 4.5 \text{ feet})$$

are different for all ages except the index age (50 years for these curves).

SITE INDEX CURVE CONSTRUCTION

1. For the site index curves, the tallest heights (HT) at each decade were read from the freehand curves and related to the site index (SI) for each plot by the equation,

$$SI - 4.5 \text{ feet} = a + b (HT - 4.5 \text{ feet}). \quad (2)$$

The following estimates were obtained:

Breast-high age (years)	<u>a</u>	<u>b</u>	<u>r</u> ²	Standard error of the estimate	Number of observations
10	32.2881	3.2310	0.3803	13.09	32
20	17.2388	1.8723	.7681	8.01	32
30	9.3532	1.3814	.9096	5.00	32
40	4.3867	1.1326	.9713	2.82	32
50	0	1	1	0	32
60	-1.7380	.8978	.9831	2.17	30
70	-1.1413	.8042	.9519	3.73	28
80	1.6845	.7048	.9498	3.98	18
90	-3.0283	.7101	.9489	3.96	10
100	-2.6900	.6767	.9399	4.03	10

The 10 sample plots with a bh age of 90 or more years have site indexes of 54.3, 55, 65.2, 69, 69.2, 81.8, 91, 91.6, 95, and 97.4 feet.

2. The above decadal estimates of b were smoothed over age (fig. 4) by the equation (forced though a b value of 1 at a breast-high age of 50 years),

$$\hat{b} = 0.52032 - 0.0013194 \text{ age} + 27.2823/\text{age};$$

where age here and in the equations to follow is breast high age. The standard error⁵ and R² values for this equation are 0.0213 and 0.9994, respectively.

⁵Standard error in this paper used with nonlinear equations is equal to $(\sum (y - \hat{y})^2 / (n - k))^{0.5}$; where y and $\hat{y}$ are actual and predicted values, n is the number of points used to fit the curve, and k is the number of parameters that have been estimated in fitting the regression (Snedecor and Cochran 1967, p. 385).

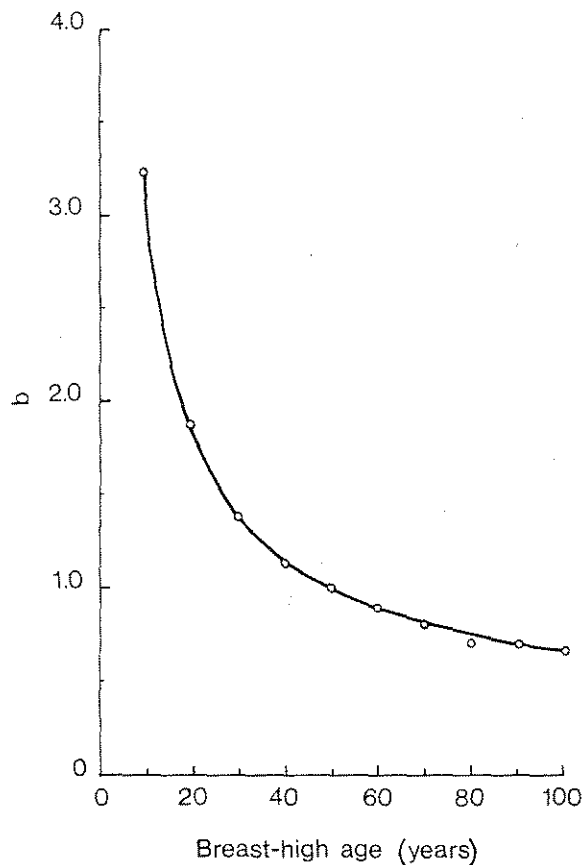


Figure 4.-- b values in equation $SI - 4.5 \text{ feet} = a + b (HT - 4.5 \text{ feet})$ as a function of age. Points are actual b values. Solid line is curve expressed by the equation, $\hat{b} = 0.52032 - 0.0013194 \text{ age} + 27.2823/\text{age}$.

The resulting $\hat{b}$ values are those appearing in table 1.

3. The following equation (with a standard error of 0.54 feet⁶ and an R^2 of 0.9999), expressing decadal mean heights as a function of age, was conditioned to pass through mean site index ($SI = 84.47$) at 50 years (fig. 5):

$$\hat{HT} - 4.5 = e^{(-0.37496 + 1.36164(\log_e \text{age}) - 0.00243434(\log_e \text{age})^4)}.$$

Here $\hat{HT}$ is an estimate of $\overline{HT}$. At ages beyond 50 years, the sample became progressively smaller and mean site index was slightly different. Average heights were adjusted to the mean overall site index using a_1 and b_1 values of the individual regressions of

$$HT - 4.5 = a_1 + b_1 (SI - 4.5)$$

for each decade in the equation,

$$\text{Adjusted average height} - 4.5 \text{ feet} = a_1 + b_1 (84.47 - 4.5)$$

before fitting the average height curve.

⁶These standard errors and R^2 values are for the equation as written, not its logarithmic form.

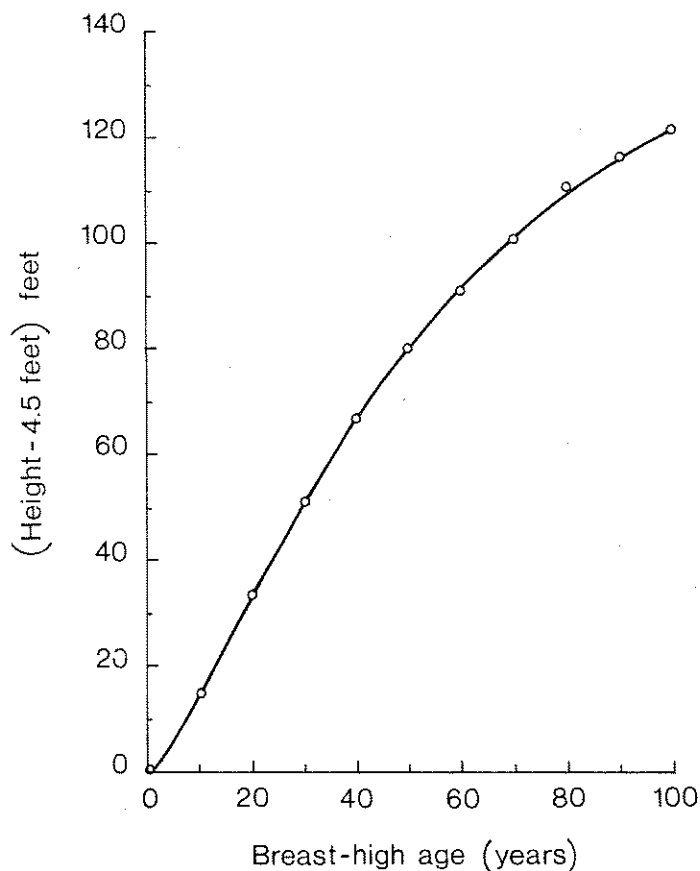


Figure 5.--Average height of sectioned trees as a function of breast-high age. Points are average heights - 4.5 feet. Solid line is curve expressed by the equation,

$$\overline{HT} - 4.5 \text{ feet} = e^{(-0.37496 + 1.36164 (\log_e \text{age}) - 0.00243434 (\log_e \text{age})^4)}.$$

4. HT and the smoothed slope b of regressions for each year were then used to calculate the corresponding intercept a :

$$\hat{a} = \bar{SI} - 4.5 - \hat{b} (\hat{HT} - 4.5).$$

These a values appear in table 1.

5. Substituting expressions for a , b , and HT in the basic equation of step 1 gives the final equation used to estimate site index as a function of breast-high age and height (fig. 3).

$$SI = 84.47 - \left[e^{(-0.37496 + 1.36164 (\log_e \text{age}) - 0.00243434 (\log_e \text{age})^4)} \right] \\ \cdot (0.52032 - 0.0013194 \text{ age} + 27.2823/\text{age}) \\ + (HT - 4.5)(0.52032 - 0.0013194 \text{ age} + 27.2823/\text{age}).$$

HEIGHT GROWTH CURVE CONSTRUCTION

1. For height growth curve construction, the site indexes for each plot were related to the tallest heights at each decade by the equation,

$$HT - 4.5 = a_1 + b_1 (SI - 4.5);$$

the following estimates were obtained:

Breast-high age (years)	a_1	b_1	r^2	Standard error of the estimate	Number of observations
10	5.3429	0.1177	0.3803	2.50	32
20	.6967	.4103	.7681	3.75	32
30	-1.5405	.6584	.9096	3.45	32
40	-1.8455	.8576	.9713	2.45	32
50	0	1	1	0	32
60	3.4273	1.0950	.9831	2.40	30
70	6.1089	1.1837	.9519	4.53	28
80	2.8416	1.3477	.9498	5.51	18
90	9.5137	1.3358	.9486	5.43	10
100	10.4112	1.3889	.9399	6.13	10

2. The above decadal estimates of b_1 were smoothed over age (fig. 6) by the equation,

$$\hat{b}_1 = -0.2828 + 1.87947(1 - e^{-0.022399 \text{ age}})^{0.966998};$$

the resulting $\hat{b}_1$ values are those appearing in table 2. The standard error and R^2 values are 0.0278 and 0.9969 for this equation which was forced through a b_1 value of 1 at age 50 years. These standard error and R^2 values are not measures of variation within the sampled population; they are given merely to show how well the fitted equation described the actual slope values for each decadal age.

3. The same expression for decadal height used in determining site index was used again with the mean site index ($SI = 84.47$) in a rearrangement of the basic equation,

$$\hat{a}_1 = HT - 4.5 - \hat{b}_1 (\bar{SI} - 4.5)$$

to produce the a_1 values shown in table 2.

4. Appropriate rearrangement and substitution for a_1 , b_1 , and HT in the basic equation give the final equation used to estimate height as a function of age and site index as shown in figure 2:

$$\begin{aligned} HT = & 4.5 + e^{(-0.37496 + 1.36164(\log_e \text{age}) - 0.00243434(\log_e \text{age})^4)} \\ & - 79.97 (-0.2828 + 1.87947 (1 - e^{-0.022399 \text{ age}})^{0.966998}) \\ & + (SI - 4.5) (-0.2828 + 1.87947 (1 - e^{-0.022399 \text{ age}})^{0.966998}) \end{aligned}$$

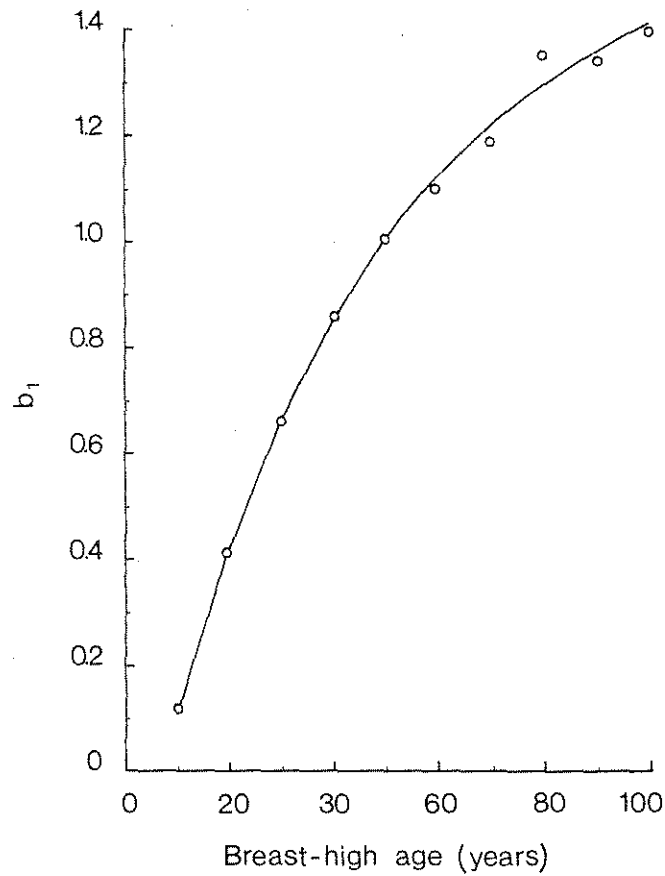


Figure 6.-- b_1 values in the equation $HT - 4.5$ feet = $a_1 + b_1$ (SI - 4.5 feet) as a function of age. Points are actual b_1 values. Solid line is expressed by the equation,

$$\hat{b}_1 = -0.2828 + 1.87947 (1 - e^{-0.022399 \text{ age}})^{0.966998}.$$

A graphic comparison between site index and the height growth curves is shown in figure 7.

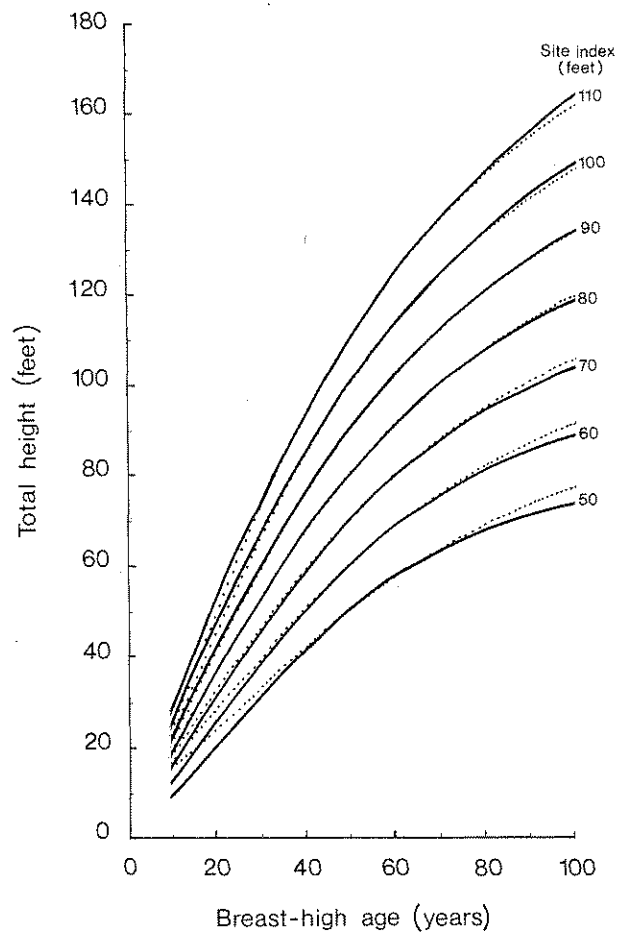


Figure 7.--Site index (solid lines) and height growth curves (dashed lines) for managed, even-aged stands of Douglas-fir east of the Cascades in the Pacific Northwest.

The final estimating equations for both site and height fit the basic data regression points well (figs. 8 and 9).

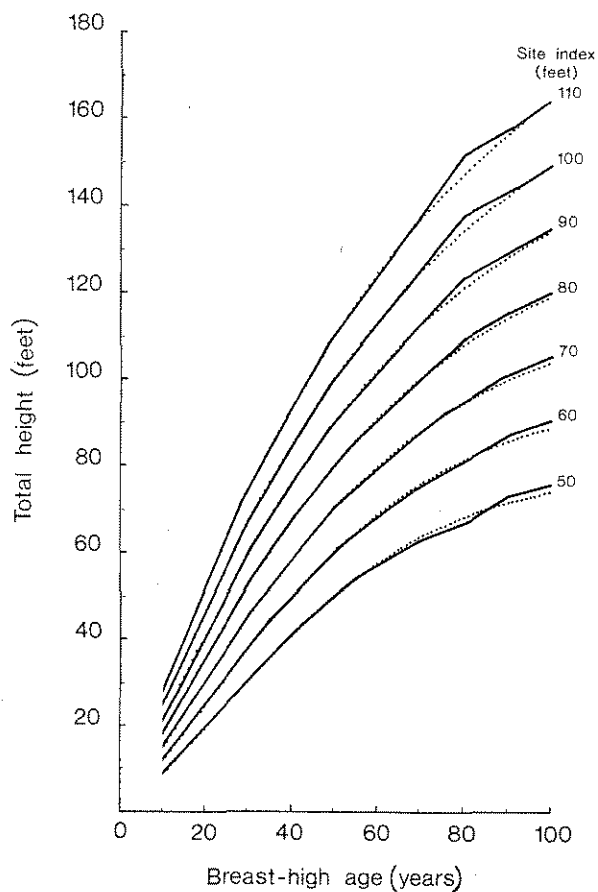


Figure 8.--Site index curves for managed, even-aged stands of Douglas-fir east of the Cascades in the Pacific Northwest. Solid lines connect decadal points derived from the unsmoothed basic data regressions of the equation, $SI - 4.5 = a + b (HT - 4.5)$. Dashed lines represent smooth curves from the following rearrangement of the estimating equation,

$$\begin{aligned}
 HT = & 4.5 + ((SI - 84.47) + (0.52032 \\
 & - 0.0013194 \text{ age} + 27.2823/\text{age})) \\
 & (e^{(-0.37496 + 1.36164(\log_e \text{age})} \\
 & - 0.00243434(\log_e \text{age})^4) / (0.52032 \\
 & - 0.0013194 \text{ age} + 27.2823/\text{age})).
 \end{aligned}$$

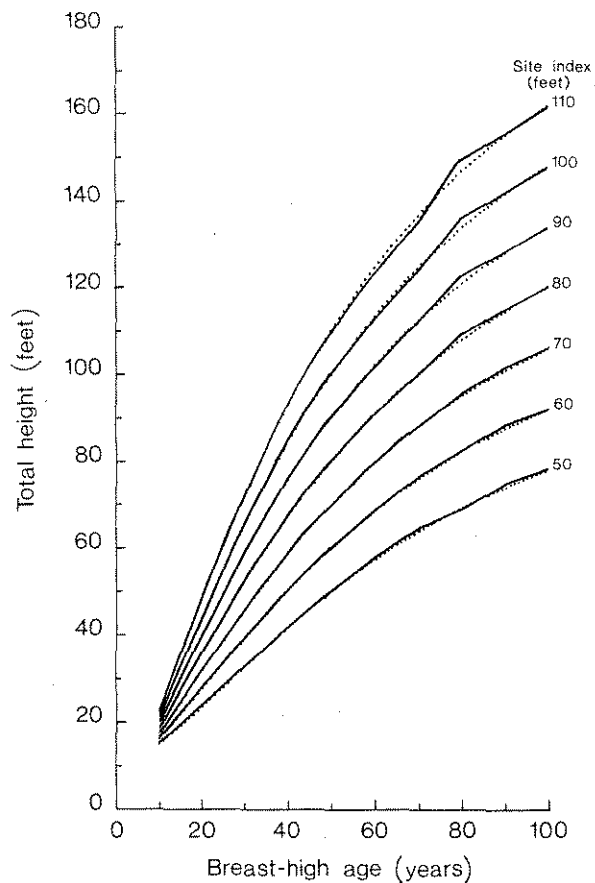


Figure 9.--Height growth curves for managed, even-aged stands of Douglas-fir east of the Cascades in the Pacific Northwest. Solid lines connect decadal points derived from unsmoothed basic data regression of the equation,

$$HT - 4.5 = a_1 + b_1 (SI - 4.5).$$

Dashed lines represent smooth curves from the estimating equations.