

# *Hydrologic modeling in the Coniferous Forest Biome*

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## *Abstract*

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*The objective of the hydrology program is to prepare a model which will provide predictions of the hydrologic state of a coniferous watershed at any desired time and in any desired place, where state is defined by the input needs of the other submodels or systems, particularly the producer and biogeochemical processes. Subsurface flow is the dominant runoff mechanism in coniferous watersheds and one of the least understood processes in hydrology. Research projects in hydrology seek to understand this process using three different techniques. One project relates subsurface flow to soil properties using direct measurement techniques. Another project approaches the problem using simulation techniques. The third project utilizes systems analysis and statistical decomposition of runoff events to make inferences about subsurface flow. These studies of hydrologic processes will be incorporated into a hydrologic model and linked to studies of other systems. The first step in linking our model with those of other groups is watershed stratification, a problem now solved by our modeling efforts.*

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## *Introduction*

Water is an essential component of any ecosystem. In the Pacific Northwest, water is a dominant element. The coniferous forests of this region are noted as some of the best-watered terrestrial ecosystems in the United States; water is the major linkage which ties the terrestrial portion of the coniferous ecosystem to the aquatic portion.

Water performs several functions which

foster this linkage. Water must be viewed as a carrier. It carries organic and inorganic nutrients between the several compartments of the terrestrial portion of the ecosystem and from the terrestrial to the aquatic portion. Water also carries sediment from the terrestrial to the aquatic portion of the system.

Water must also be viewed as a nutrient itself. It is an essential component of most biologic processes. The availability of water in the soil governs both the initiation and termi-

nation of any process as well as the rate at which it proceeds.

### Objective of the Hydrology Program

The objective of our program is to prepare a model which will provide predictions of the hydrologic state of a coniferous watershed at any desired time and in any desired place, where state is defined by the input needs of the other submodels or systems, particularly the producer and biogeochemical processes.

### Structure of the Hydrology Modeling Effort

Our modeling effort is organized to pay particular attention to the many functions of water in the forest ecosystem. A generalized model for water flow through a forest system was conceptualized long ago. This model is often called the hydrologic cycle. Rothacher et al. (1967) showed for the H. J. Andrews Experimental Forest that water movement in the forest soil and evapotranspiration are the most significant processes governing water flow.

Our studies focus upon subsurface water movement. One project measures subsurface flow directly in the study watershed. Another study is a computer simulation of a watershed. This approach will provide yet another avenue for assessing soil moisture and the subsurface flow of water. The simulation model will be calibrated using 14 years of record on watershed 2. Then the model will be verified with the data available on watershed 10. A third project seeks to develop techniques for predicting the subsurface flow component using a systems analysis technique for statistical decomposition of the hydrograph into its components. Other studies will contribute submodels to the simulation of the hydrologic system. The work of the Primary Producer group on a transpiration model as well as the work of the Meteorology group on an evapotranspiration model will aid our modeling effort significantly.

We are concurrently working toward a more spatially refined model, the character of

which is determined as much by biologic as hydrologic constraints. Our latest efforts have focused upon devising a system for stratification of watershed 10 which is amenable to both hydrologic and biologic models.

Other hydrology projects are included in the biome effort. We hope to provide hydrologic measurements as a part of the work at Findley Lake and the evapotranspiration study at the Thompson site. We shall soon begin to solicit and organize available data from coniferous watersheds throughout the West in anticipation of extrapolating our model to other ecosystems.

This has provided a broad overall view of the Coniferous Biome Hydrology Program—its structure to achieve a better understanding of water flow through the ecosystem and its interaction with other system components. A more detailed description of the hydrology efforts follows. Each project focuses upon evaluating water flow in a forested watershed. Analytical techniques vary between projects, but the ultimate goal of modeling subsurface flow mechanisms and watershed response links all projects together.

## *Subsurface Movement of Water on Steep, Forested Slopes<sup>1</sup>*

With the exception of stream channel interception, the hydrographs of watersheds in the forested, steep topography of western Oregon reflect overall subsurface movement of water. Watershed response is rapid but without surface runoff (Barnett 1963, Rothacher et al. 1967). Although subsurface flow is by far the major component of the hydrograph, virtually nothing is known about the process on steep slopes. The objective in our study is to characterize the subsurface movement of water in steeply forested topography.

<sup>1</sup> Authored by R. Dennis Harr, Assistant Professor, Oregon State University, Corvallis.

The study areas are located at several of the lower watersheds in the H. J. Andrews Experimental Forest near Blue River, Oregon. Vegetation is typical of the low-elevation Douglas-fir forest. Study slopes average about 75 percent. Soil depth is variable with maximum depths in excess of 5 meters. Because of high porosities (70-80 percent) and large proportions of macropores, these soils drain rapidly. Permeabilities of 5,000 and 900 mm per hour have been noted on nearby watersheds for surface soil and subsoil, respectively (Dyrness 1969).

### Methods

Initial investigations are being directed toward describing the physical properties of the porous medium through which water moves on its way to a stream. These field and laboratory investigations will indicate where water movement most likely occurs.

Drilling with a portable power drill will follow a grid pattern over a small stream-to-ridge portion of slope. At each grid point soil depth, depth and thickness of saprolite, and depth to unweathered bedrock will be determined. Additional drilling between initial grid points will indicate in more detail the surface contour of the impermeable parent material. Aluminum tubing placed in each hole will provide access for measurement of ground-water level or soil moisture content.

In the laboratory, undisturbed soil cores taken from various depths in soil pits located over the study area are being analyzed. Such properties as porosity, pore-size distribution, stone content, permeability, and moisture retention characteristics are being evaluated.

The type and amount of measurements to be made during and following winter storm events in 1972-73 will depend on the information gathered during initial field and laboratory investigations now underway. Anticipated measurements include soil moisture content, vertical and lateral extent of saturated flow, soil moisture tension, precipitation, and water outflow from the base of the slope. Drilling and tracer studies will attempt to define the source area for this

water.

### Preliminary Results

Although the study has just recently begun, certain observations have provided qualitative information concerning the subsurface flow process on steep slopes. Precipitation moves downward under the influence of gravity until this movement is obstructed. In some parts of the study area this obstruction may be caused by rock fragments which cause shallow, localized saturation as evidenced in several soil pits during a period of heavy rain. Where rock fragments are not present, downward movement of water continues until the relatively impermeable parent material is reached. Here saturation occurs, flow acquires a horizontal component, and water begins moving toward the stream.

At some point on the slope this saturated flow is concentrated into pipelike subsurface channels. The cause of this concentration is unknown but could conceivably result from the microrelief of the impermeable material, from bedrock fractures, or from decayed root channels. At the toe of the slope the channels are spaced about 1-6 meters apart. They lie on the bedrock surface and appear associated with surface micro-relief. Shapes of their cross sections range from circular to flat rectangular. Width is also variable, ranging from 1 centimeter to about a meter. Where these channels discharge into the stream channel, they are separated by soil which may contain a shallow saturated lower layer from which seepage occurs. Water velocity of the seepage appears to be several orders of magnitude lower than that of the subsurface channels. The latter accounts for the greatest portion of stormflow.

The subsurface channels evident at the toe of the study slopes may be outlets of a subsurface drainage system much like that described for other humid areas (Jones 1971). Water can be observed discharging from such channels in roadcuts and recent soil slumps at various slope positions in the vicinity of the study area. Such a subsurface drainage system could account for the rapid hydrologic response of these steep slopes.

# Computer Simulation of Forest Watershed Hydrology<sup>2</sup>

A hydrologist is often faced with the need to predict system responses under various possible management alternatives. One approach to this problem is to apply the technique of computer simulation, whereby a quantitative mathematical model is developed for investigating and predicting the behavior of the system. In this study, a computer model is being developed to simulate the hydrologic responses of a forest watershed, emphasizing the measurable variables related to the plant communities and soil types of the watershed. The model represents the inter-related processes of the system by functions which describe the different components of physical and biological phenomena in a watershed.

## Scope and Objectives of the Simulation Study

In the first phase of the study, the scope is being limited to the formulation of a fundamental model of watershed hydrology which takes precipitation as the basic input and evapotranspiration and streamflow as outputs of the system. The various component processes within the system are linked by the conservation of mass principle. Depending upon energy levels, water can vary among its solid, liquid, and vapor forms; hence, the energy budget is used as an auxiliary tool for maintaining the water balance. That aspect of the system involving water as a carrier of nutrients and sediments will be examined in a subsequent phase of the study. Under this next phase a water quality submodel will be formulated and added to the quantity model now being developed.

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<sup>2</sup> Authored by J. Paul Riley, Professor, Utah Water Research Laboratory, Utah State University, Logan; George B. Shih, Research Engineer, Utah Water Research Laboratory, Utah State University, Logan; and George E. Hart, Associate Professor, Forest Science, Utah State University, Logan, Utah.

The specific objectives of the current phase of the study are stated as follows:

1. To develop and verify (calibrate and test) a hydrologic simulation model for a small forested subwatershed on the H. J. Andrews Experimental Forest.
2. To estimate through model sensitivity studies the relative importance of various processes within the hydrologic system of the model, with particular emphasis on evaluating the soil moisture and inter-flow components.

## Hydrologic System Models

Several hydrologic simulation models are currently available. Examples which might be cited include Crawford and Linsley (1964), Sittner et al. (1969), and Riley et al. (1966). However, in order to meet the needs of this study all existing models require some modifications and further development. Therefore, on the basis of previous work at Utah State University a computer model is being developed to simulate the hydrologic behavior of forest watersheds. The model will be applicable to a wide variety of geographical areas and management problems. In this study, data from watershed 2 on the H. J. Andrews Experimental Forest will be used to demonstrate the utility of the model. Figure 1 summarizes the geophysical features of the study area (Rothacher et al. 1967). Data requirements include air temperature, precipitation, runoff hydrographs, and characteristics of the watershed (average slope, degree and aspect, vegetative cover, density of vegetative cover, soil moisture holding characteristics, and drainage density). Other observed records, such as snow depth, soil moisture content will be used to check the performance of the model in simulating various component processes of the system.

Figure 2 illustrates the various component processes represented in this model, with the boxes representing storage locations and the lines transfer functions. Under this study, the hydrology of the drainage area will be synthesized first as a lumped parameter model in which the entire watershed area is considered as a single space unit. On this basis, a dis-

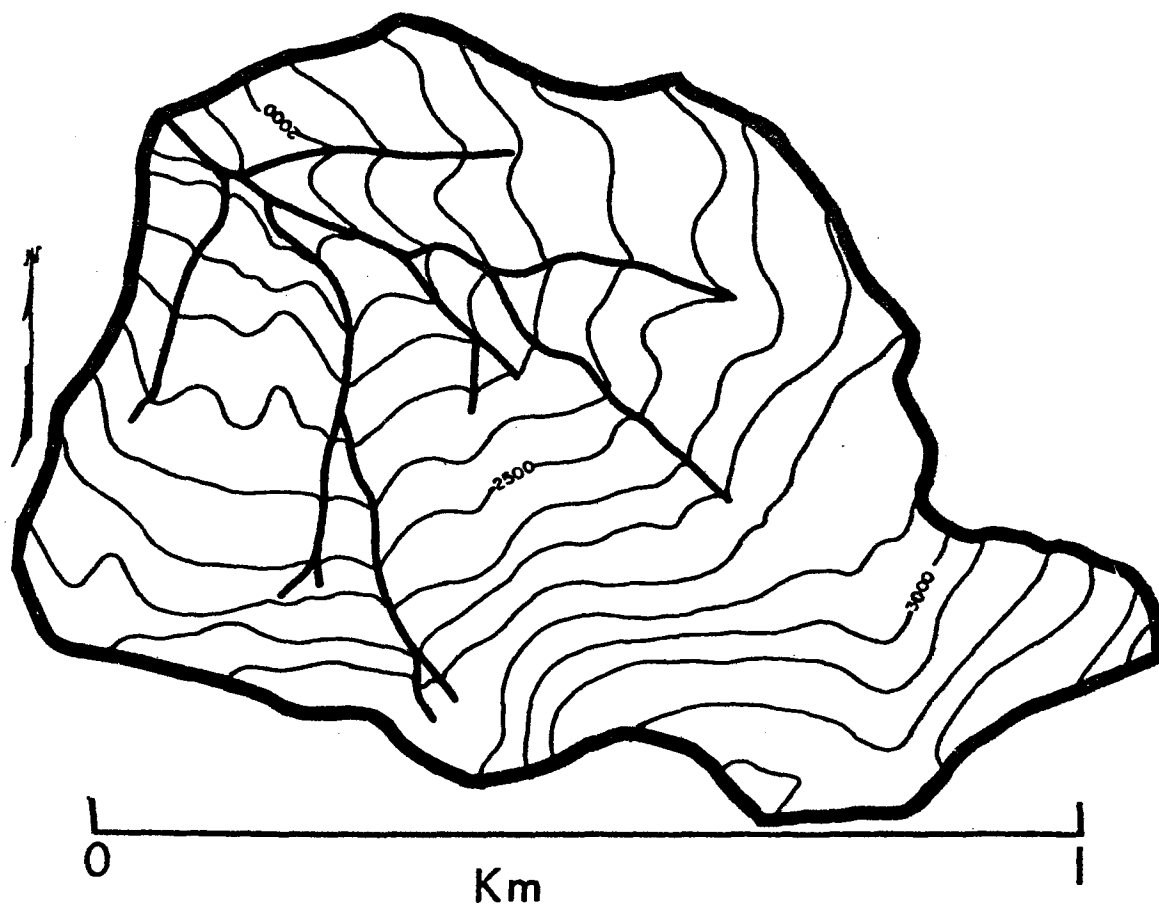


Figure 1. Watershed 2, H. J. Andrews Experimental Forest, Oregon. Area: 60.3 hectares; aspect: NW; average slope (percent): 61.1; elevation min.: 526 m; elevation max.: 1,078 m; main channel length: 1,108 m; drainage density: 4.3 km/km<sup>2</sup>; precipitation (1952-62): 2,400 mm/yr; runoff (1953-62): 1,560 mm/yr; average evapotranspiration (1959-62): 540 mm/yr.

tributed parameter model will be developed in which the watershed will be divided into four space units, roughly corresponding to sub-watersheds within the area. The model will compute continuous daily streamflow for each subarea and route the contribution of each down the streams to the gaging station, where the computed and observed discharges will be compared. Other important output functions from the model will include soil moisture, actual evapotranspiration, and snow depth. Several of the component processes which are illustrated by figure 2 are discussed in the following sections.

#### Interception

Interception is the part of precipitation

that is caught temporarily by forest canopies and then redistributed either to the atmosphere by evaporation or sublimation or to the forest floor. The amount of interception depends upon storm size and intensity, and canopy type and density. A report by Rothacher (1963) showed that throughfall in the study area was related to storm size by the equation:

$$\text{Throughfall} = 0.8311 \times (\text{gross precipitation}) - 0.117 \quad (1)$$

In equation 1 throughfall is, of course, bounded by the condition that it must be greater than or equal to zero. Although larger amounts of snow may be temporarily intercepted than rain, there is strong evidence that most intercepted snow ultimately falls and becomes part of the snowpack.

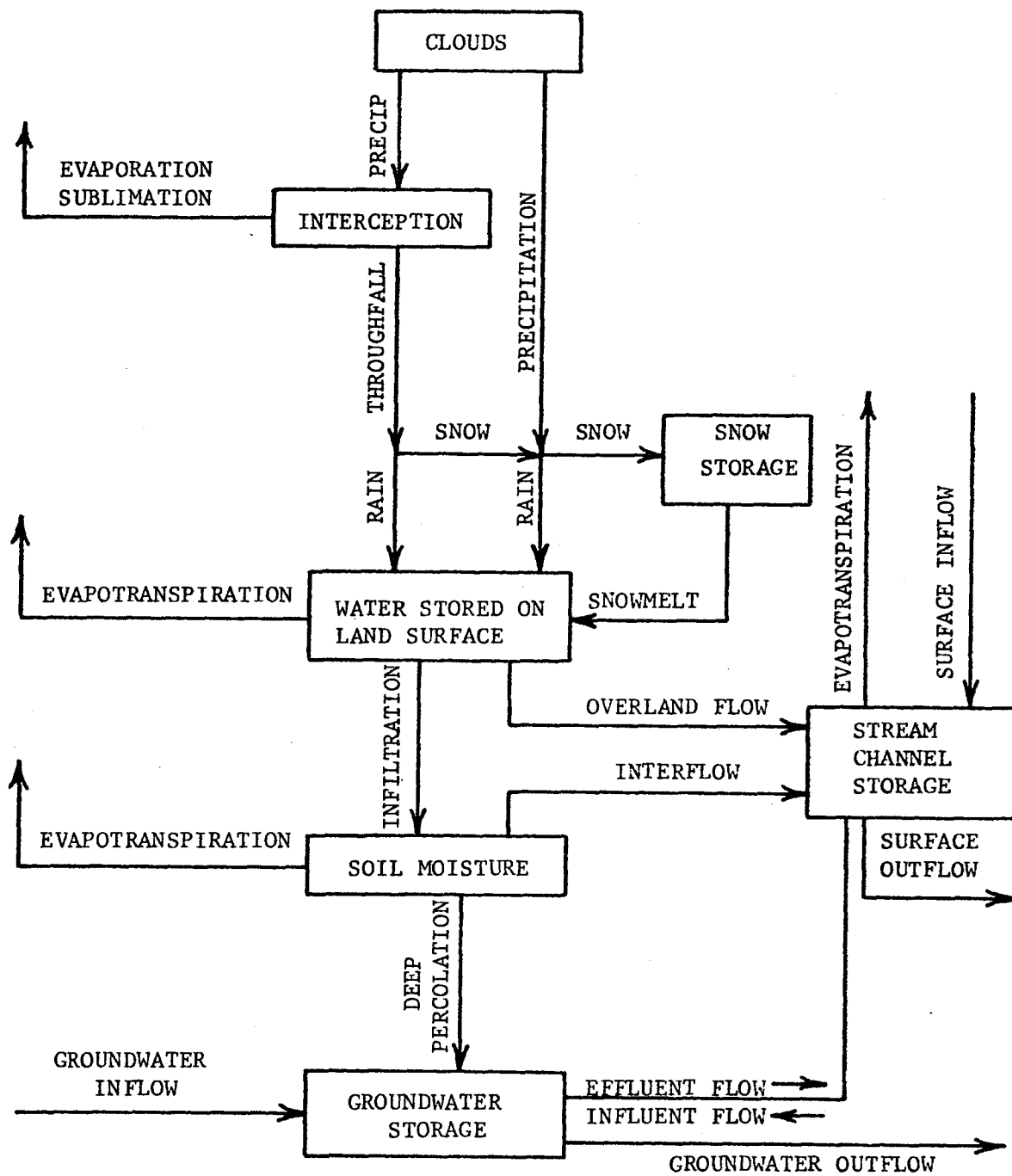


Figure 2. A flow diagram of the hydrologic system within a typical watershed area.

An alternative way of considering interception quantities in a model is to express interception rate as a decaying function of time limited by an average interception storage capacity for the watershed canopy. This approach was incorporated into a watershed simulation model by Riley et al. (1966).

### Snow Storage and Melt

#### Forms of Precipitation

Only two forms of precipitation, rain and snow, are considered in this study, with a surface air temperature criterion being applied to establish the occurrence of these two forms. Figure 3 (U.S. Army Corps of Engineers 1956) shows that at a temperature of 1.5°C there is a 50-percent chance that the precipitation will be in the form of snow. A straight-line fit to figure 3 is used to deter-

mine the portion of rain in a given day according to the following equation.

$$R = P \cdot \frac{T_a - T_s}{T_r - T_s} \quad (2)$$

in which

- R = estimated portion of total daily precipitation occurring as rain
- P = total daily precipitation
- $T_a$  = mean daily surface air temperature
- $T_s$  = mean daily air temperature below which all precipitation is assumed to occur as snow
- $T_r$  = mean daily air temperature above which all precipitation is assumed to occur as rain

Precipitation falling as snow will be accumulated on the watershed until air temperatures rise sufficiently above the freezing point to initiate snowmelt.

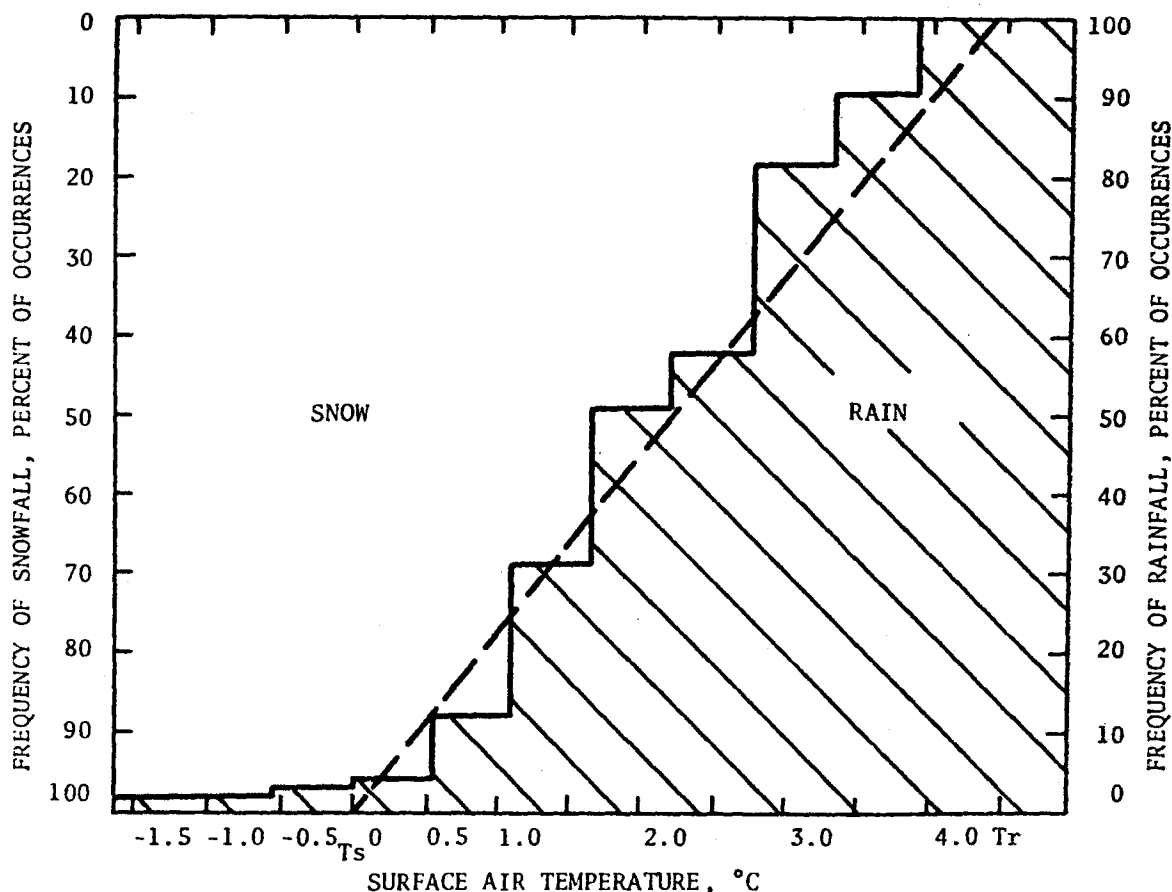


Figure 3. Frequency distribution of precipitation in rain and snow forms.

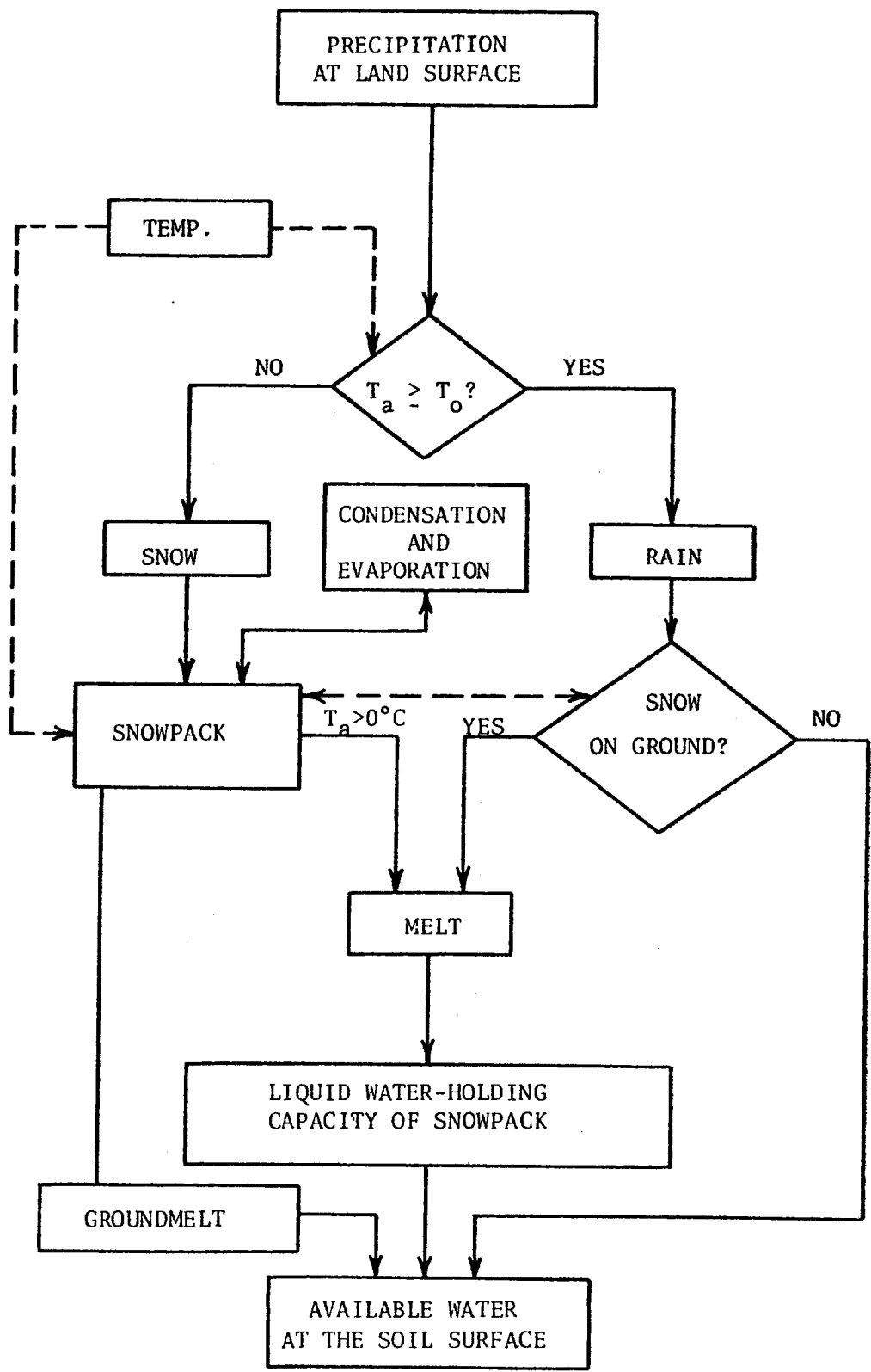


Figure 4. A flow chart of the snow accumulation and ablation processes.



## Snowmelt

A flow chart of the snow accumulation and ablation processes is shown by figure 4. Rate of snowmelt depends primarily upon the rate of energy input to the snowpack. However, both the complex nature of snowmelt and data limitations prevent a strictly analytical approach to the simulation of this process, and air temperatures are frequently applied as an index of available energy. Examples of researchers who have used this approach are Pysklywec et al. (1968), Anderson and Crawford (1964), Amorocho and Espildora (1966), and Eggleston et al. (1971). Because temperature data are the only indicators of energy levels available on watershed 2, a degree-day approach based upon the work of Eggleston et al. (1971) will be used in the model of this study to represent the snowmelt process at the surface of the snowpack. This component submodel includes mathematical relationships for various phenomena involved in the snowmelt process. The submodel is applicable to any geographic location by determining appropriate constants for certain relationships through a verification procedure. The relationship for surface melt rate is expressed as follows:

$$M_{rs} = k_m k_v \frac{RI_s}{RI_h} T_a (1 - A) + T_a \frac{P_{rg}}{80} \quad (3)$$

in which

- $k_m$  = a constant of proportionality
- $k_v$  = vegetation transmission coefficient for radiation
- $RI_h$  = radiation index for a horizontal surface at the same latitude as the particular watershed or zone under study
- $RI_s$  = radiation index for a particular watershed zone possessing a known degree and aspect of slope
- $T_a$  = surface air temperature in °C
- $A$  = albedo, or reflectivity, of the snowpack surface
- $P_{rg}$  = precipitation reaching the snow surface in the form of rain, in centimeters

## Infiltration

Rates of water supply on the ground surface, whether in the form of rainfall minus interception or snowmelt, must exceed infiltration rates before any surface runoff occurs. The infiltration rate depends on the physical and moisture characteristics of the soil, as well as the surface organic conditions, and it is often expressed in the form of Horton's exponential equation. However, the soils of watershed 2 are very porous and no overland flow has been observed. Thus, all precipitation reaching the ground surface is assumed to infiltrate into the soil and to move to the stream channels as subsurface flow.

## Soil Moisture

Soils on the study watershed are relatively deep and have a high porosity. Data on the physical properties of the watershed soils are available (Rothacher et al. 1967), and this information will be used to determine the soil moisture holding characteristics.

The computer model allows infiltrating water to satisfy first the available moisture holding capacity of the soil within the root zone of the forest canopy. When the available soil moisture holding capacity is reached, additional infiltration is assumed to percolate by gravitation either somewhat laterally within the root zone or downward to deeper soil zones. Water which moves laterally usually reaches a surface channel within a relatively short period of time, whereas deep percolation moves from the watershed more slowly and sustains streamflow during dry seasons. From preliminary studies (Rothacher et al. 1967), approximately 87 percent of the total annual precipitation reaches the ground surface, and about 75 percent of this quantity becomes surface runoff. From an analysis of streamflow hydrographs it is estimated that about 10 percent of the runoff comes from baseflow, which is contributed from deep percolation. Thus, of the average annual precipitation of 2,400 mm which falls on the watershed approximately 2,100 mm enter the soil, 440 mm are abstracted by evapotranspiration, and the remaining 1,660 mm leave

the watershed as surface runoff, with 170 mm of this quantity occurring as baseflow. The soil moisture content computed by the model will be checked with observed data.

### Evapotranspiration

Factors affecting evapotranspiration include temperature, solar radiation, wind, humidity, and consumptive use by plants. However, only temperature and humidity data are available for the watershed. Among the commonly used evapotranspiration equations (Veihmeyer 1964), the Penman equation is perhaps the most rational, but the data requirements are extensive. For this reason, the modified Hargreaves (Veihmeyer 1964), will be used in this study. The equation is stated as follows.

$$U = \sum K d (0.38 - 0.0038h) T_a \quad (4)$$

in which

- U = the daily potential evapotranspiration in centimeters
- d = the daily daytime coefficient dependent upon latitude
- h = the mean daily relative humidity at noon
- T<sub>a</sub> = the mean daily surface air temperature in °C
- K = a monthly consumptive use coefficient which is dependent upon plant related characteristics, such as species, growth stage, and density on the watershed

The influence of soil water on evapotranspiration has been the subject of much research and discussion. It is now generally recognized that there is some reduction in evapotranspiration rate as the quantity of water within the root zone decreases. In this study it will be assumed that evapotranspiration occurs at the potential rate through a certain range of the available soil moisture. A critical moisture level is then reached at which actual transpiration begins to lag behind the potential rate. Within this range of the available soil moisture the relationship between available water content and transpiration rate will be assumed to be virtually linear. Thus,

$$E = U, [M_{es} \leq M_s(t) \leq M_{cs}] \quad (5)$$

in which

- E = daily evapotranspiration adjusted for the influence of soil moisture levels
- M<sub>s</sub> = quantity of water stored within the root zone and available for plant use at any time, t
- M<sub>es</sub> = limiting root zone available moisture content below which soil moisture tensions reduce evapotranspiration rates
- M<sub>cs</sub> = root zone storage capacity of water available to plants

$$E = U \frac{M_s(t)}{M_{es}}, [M_{es} > M_s(t) \geq 0] \quad (6)$$

Considering the pressure effect, the total rate of gravity water storage depletion through both interflow and deep percolation is assumed to be directly proportional to the quantity of water in this form of storage remaining in the soil profile at any particular time. The interflow portion of this depletion, N<sub>r</sub>, will be expressed as follows:

$$N_r(t) = K_i \frac{d G_s}{dt} = K_i (K_g G_s(t)) \quad (7)$$

in which

- K<sub>i</sub> = interflow depletion coefficient
- K<sub>g</sub> = gravity water depletion coefficient

That is, N<sub>r</sub> = K<sub>i</sub>K<sub>g</sub>G<sub>s</sub>(0)e<sup>-K<sub>g</sub>t</sup>, in which gravity storage at time, t = 0 is represented by G<sub>s</sub>(0) and no input to G<sub>s</sub> is assumed to occur between t = 0 and any other time, t. It is estimated that on watershed 2 about 90 percent of the gravity water storage leaves the area as interflow, in which case K<sub>i</sub>K<sub>g</sub> = 0.9.

### Groundwater

Water enters groundwater storage as deep percolation from the overlying plant root zone. The rate of deep percolation, G<sub>r</sub>, is numerically equal to the total rate of gravity water depletion within the root zone less the interflow rate. Thus,

$$G_r = (1 - K_i) \frac{d G_s}{dt} \quad (9)$$

By integrating equation 9 over a specific time period the accumulated inflow to the groundwater basin,  $G_w$ , is estimated for this time period.

$$G_w = \int_0^t (1 - K_i) d \frac{G_s}{dt} dt \quad (10)$$

If the groundwater basin is considered as a linear reservoir, the outflow rate is given by the expression

$$Q_{rg} = K_b G_w \quad (11)$$

in which

$K_b$  = a coefficient which is estimated from dry season streamflow hydrographs

$Q_{rg}$  = the outflow rate from the groundwater reservoir

By combining equations 9 and 11, the net rate of storage change within the groundwater basin is derived as

$$G_r - Q_{rg} = \frac{d G_w}{dt} \quad (12)$$

By substituting equation 11 into equation 12 and rearranging terms, the following relationship is obtained.

$$\frac{d Q_{rg}}{dt} = K_b [G_r(t) - Q_{rg}(t)] \quad (13)$$

The rate of discharge from the groundwater basin as baseflow is obtained by solving equation 13 for  $Q_{rg}$ .

### Runoff

The possible sources of streamflow at any reach within a channel are overland flow (surface runoff), interflow, groundwater, and upstream input. Manning's equation is usually applied to compute overland and channel flow rates at any point. Under conditions on watershed 2, however, surface runoff does not occur, and channel routing on a daily time increment

is not significant. Therefore, runoff rates at the stream gage are given by summing the interflow and groundwater discharge rates.

### Model Verification

Model verification includes calibration of the model parameters to a particular area, testing the sufficiency of processes defined in the model, and examining the prediction performance of the model. A self-calibration subroutine will be included in the model whereby the program will search for optimal model parameter values. Under this procedure each water year is used as a unit for optimization and the objective function is to minimize the variance between observed and computed streamflow (Shih 1971). The sufficiency of processes defined in the model is reflected in the dispersion of parameter values resulting from each year of calibration. After the model is calibrated, those years of data which were not used for calibration are used to examine the confidence level of predictions by the model. A flow diagram of the model verification procedure is shown by figure 5.

### Model Parameters

Model parameters are the coefficients used in defining the processes which have not been accurately measured or which cannot be directly measured. By establishing the values of these coefficients the general model is fitted to the hydrologic system of a specific watershed. Depending upon the resolution of the model and the availability of data, the number of parameters to be calibrated may vary. In order to avoid using a large number of degrees of freedom in the calibration process and to save computation time, the number of model parameters should be kept as few as possible. In this study, the preliminary model parameters to be calibrated are interception storage capacity ( $S_I$ ), snowmelt coefficient ( $K_S$ ), soil moisture retention capacity ( $M_{CS}$ ), gravity water depletion coefficient ( $K_g$ ), and groundwater recession coefficient ( $K_b$ ).

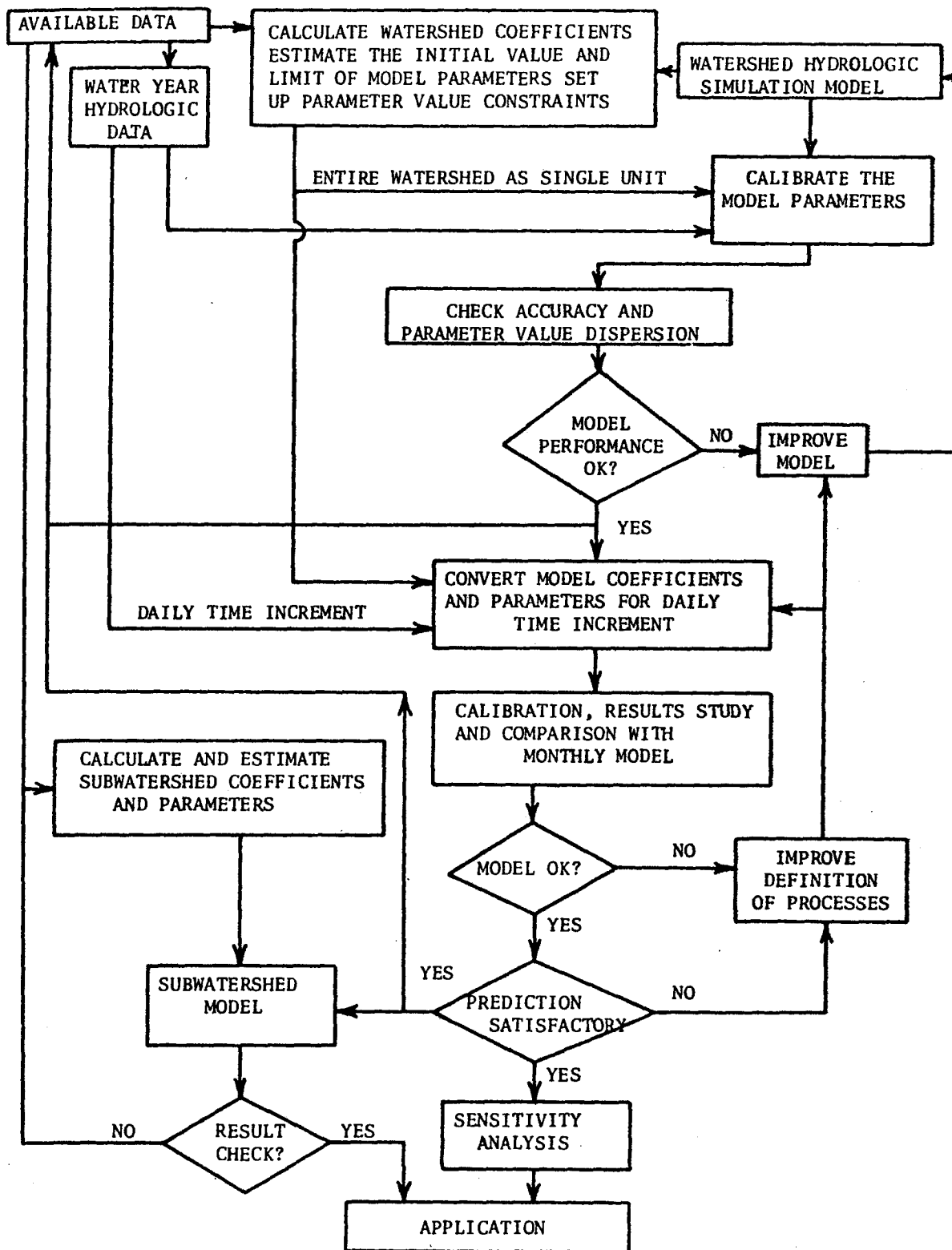


Figure 5. Flow diagram for verification procedure.

## Calibration of Parameters

In general, it is anticipated that realistic parameter values are established through the calibration procedure. However, when streamflow is the only available component for checking the model, it is possible that several combinations of parameter values will yield satisfactory agreement between observed and computed outflow hydrographs. The problem of establishing unique parameter values is approached on the basis of hydrologic judgment and by using "interior" observations, such as snow depth and soil moisture, as check points on model performance. Other ways of testing the model include the time distribution of output quantities, such as streamflow, and known (or estimated) monthly or annual quantities. For example, for watershed 2 it is estimated that interception storage is about 0.5 cm, and total interception amounts to approximately 17 percent of the annual precipitation.

Under the self-calibration technique model parameter values are altered or perturbed in a random sequence and the resulting changes in the objective function are examined (Shih 1971). A computer flow chart for the calibration subroutine is shown by figure 6. The entire program model, including the calibration subroutine, will be synthesized on a hybrid computer.

## Sensitivity and Management Studies

### Sensitivity

A sensitivity analysis is performed by changing one system variable while holding the remaining variables constant and noting the changes in the model output functions. If small changes in a particular system parameter produce large changes in the output or response function, the system is said to be sensitive to that parameter. Thus, through sensitivity analyses it is possible to establish the relative importance with respect to system response of various system processes and input functions. This kind of information is useful from the standpoint of system management, system modeling, and the assignment of priorities in the collection of field data. Under this study,

the verified model will be used to perform various sensitivity analyses for the hydrologic system of watershed 2.

### Management

Opportunities for management of a forest watershed are widely varied, and range from changes in logging practices to forms of soil treatment. Actual implementation of a management scheme depends upon benefits gained as compared with possible disbenefits. The simulation model developed under this study will not make direct comparisons of benefits and disadvantages, but will predict changes in the system output associated with given management alternatives. Under this study the capability of the model will be demonstrated for rapidly testing many possible management alternatives.

Much of the work discussed by this paper is based upon past developments in watershed simulation at Utah State University. Hydrologic modeling of the H. J. Andrews Experimental Forest for Coniferous Forest Biome has just begun, and the preceding discussion has been influenced by a consideration of particular conditions in the study area. However, the model will be fundamental in concept, and therefore generally applicable in a geographic sense. Whenever feasible, the model will use basic equations which are valid for short time intervals to define the various processes in the model. The output will then be summed for application to daily or longer time increments. For example, Horton's infiltration equation is applicable in minute units, but by summing these quantities, the model is capable of calculating equivalent daily infiltration rates. For the area under this study, however, it is assumed that water supply rates at the ground surface do not exceed infiltration capacities, so that surface runoff does not occur, thus considerably simplifying the model calibration process.

System functions which are important to forest management and other aspects of the total project include evapotranspiration and soil moisture. These functions, along with streamflow, will be estimated by the model for use in other parts of the total system

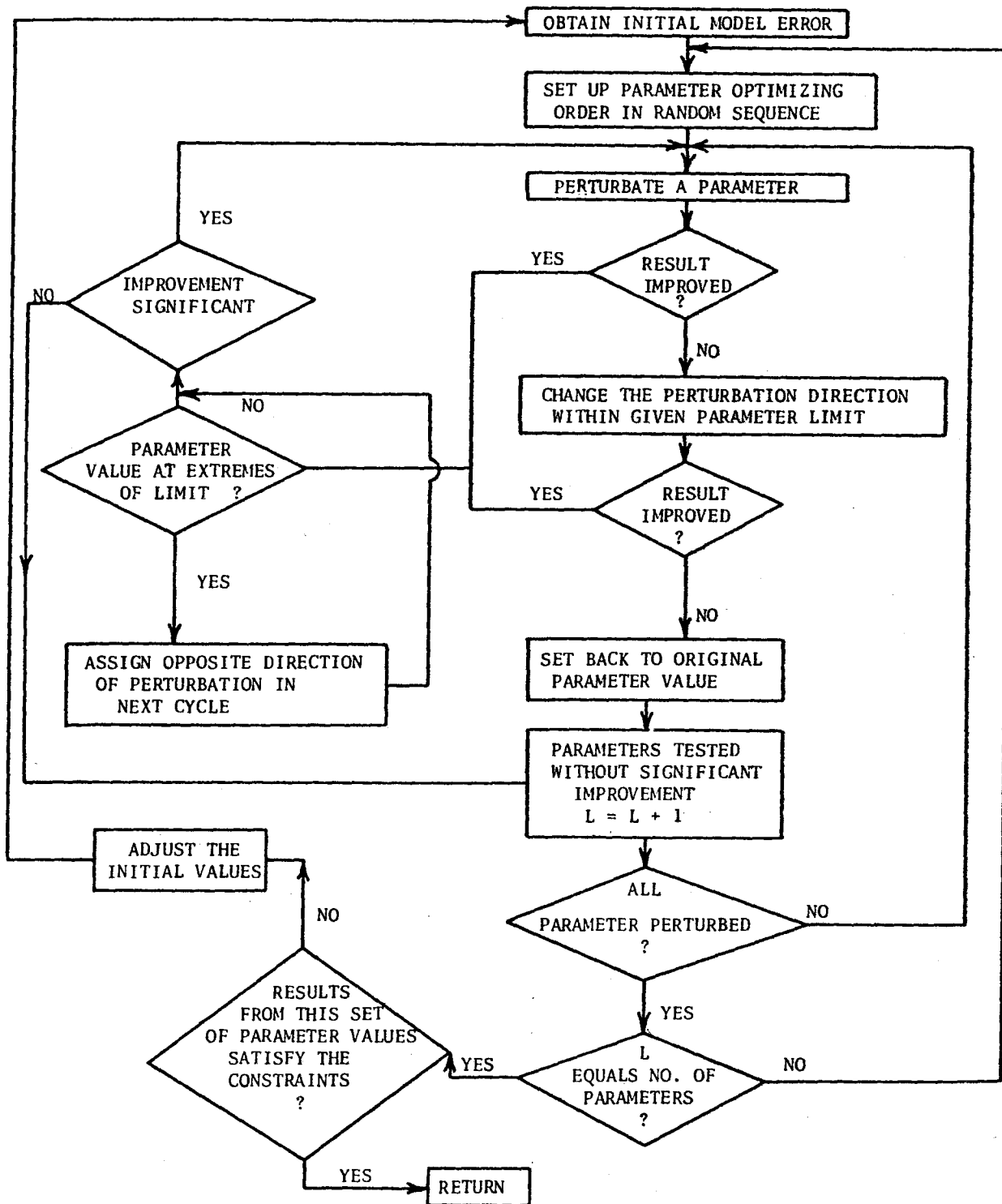


Figure 6. Flow chart for the model calibration.

model being developed under the Coniferous Biome Program. The important underlying feature throughout the entire study will be that all of the separately described hydrologic processes and phenomena are interlinked into a total system. Thus, from the model, hopefully, it will be possible to evaluate the relative importance of the various items, explore critical areas where data and perhaps theory are lacking, and finally establish guidelines for the improved management of forest watersheds.

### Hydrologic Systems Analysis<sup>3</sup>

The purpose of this research is to devise a technique for statistical decomposition of a hydrologic event such that system processes such as precipitation, subsurface flow, and evapotranspiration, which contribute to the observed streamflow can be separated and described. This technique will therefore provide one more avenue for determination of the subsurface flow process on forest soils. The technique chosen for this research is a form of systems analysis.

#### Systems, Definitions and Basic Principles

System may be defined as an aggregate of physical parts that do not change with time, operating on an input to produce an output, both being functions of time. The simplified representation of a watershed, given in figure 2, can be considered as a system whose input is precipitation and runoff its output. System "synthesis" is a technique employed when the system is known in terms of a mathematical equation; the objective is to determine the nature of the output for any class of input (fig. 7). In system "analysis" a system response function or kernel which best describes a given input-output pair is derived (fig. 7). The term "best" implies that the derived kernels are not unique. Combinations of both techniques can be used for the solution of hydrologic problems. A system can

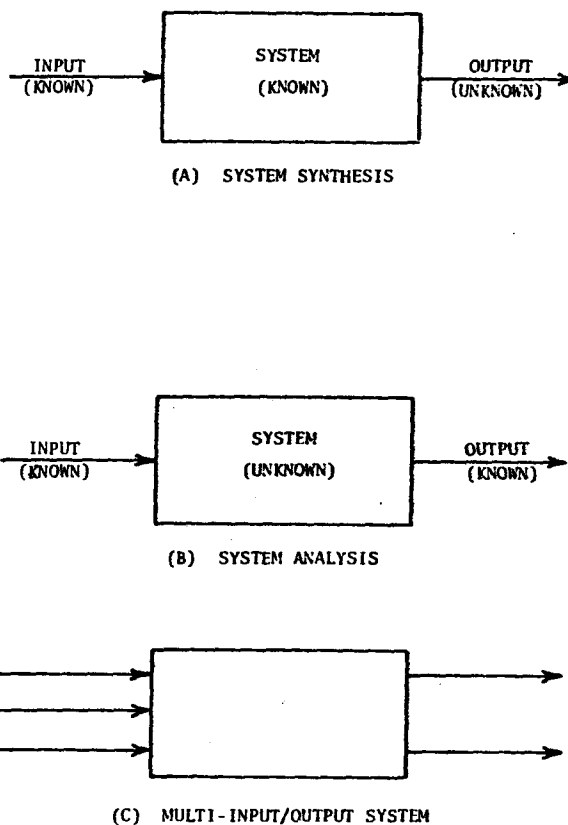


Figure 7. Illustration of systems.

have one input and one output or many inputs and outputs (fig. 7). A system is "lumped parameter" if input and output are functions of a single variable. Otherwise, the system is of the "distributed parameter" type. If the system response at any time, due to a given input, is uniquely determined, the system is said to be "deterministic." If the system response is subject to uncertain influences, the system is "stochastic or probabilistic."

A quantity  $z$  is a "functional" for the function  $x(t)$  in the interval  $(a,b)$ , if it depends upon all values taken by  $x(t)$ , when  $t$  varies in the interval  $(a,b)$ . An illustration of a functional is given in figure 8. The output of a system is a functional of the input and, for the same reason, runoff is a functional of precipitation. A system is "time invariant" if it does not change with time. Such systems can be represented by functionals. "Physically realizable" is a system whose output at time  $t$  depends only upon past values of the input.

<sup>3</sup> Authored by Z. G. Papazafiriou, Research Associate, and R. H. Burgy, Professor, University of California, Davis.

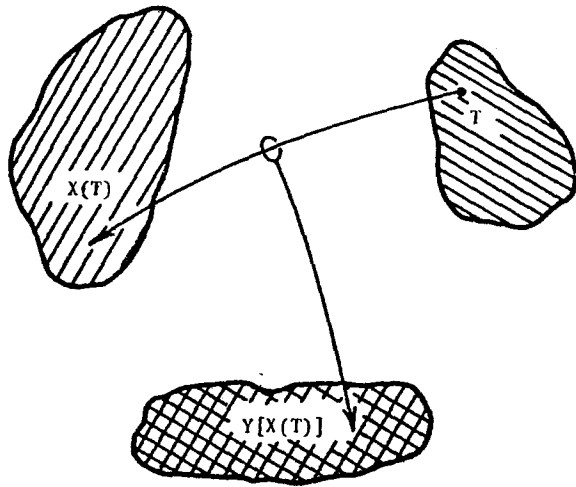


Figure 8. Demonstration of a functional.

Hydrologic systems are physically realizable since their outputs (runoff) at time  $t$  depend only on the past values of their inputs (precipitation). The "memory" of a system is the time period between some past time and the present for which the output depends only upon the input. If the output depends only on the present value of the input, the system is said to be a "no-memory" system. If the output of a time invariant system is analytic about zero input at some time  $t_0$ , the system is "analytic". Analyticity is very important, since if a system is analytic, its output can be expanded in Volterra series. Hydrologic systems are assumed to be analytic.

A deterministic system  $\underline{H}$  is said to be "linear," if given the inputs  $X_1(t)$  and  $X_2(t)$  such that

$$y_1(t) = \underline{H} [AX_1(t)] \quad (14)$$

$$y_2(t) = \underline{H} [BX_2(t)] \quad (15)$$

implies that

$$y_1(t) + y_2(t) = y[X(t)] \quad (16)$$

$$= \underline{H} [AX_1(t) + BX_2(t)]$$

$$= \underline{AH} [X_1(t)] + \underline{BH} [X_2(t)]$$

that is, in a linear system, each member of a sequence of input values influences the output independently of every other. This is the well known principle of superposition. If a system does not satisfy the above condition it is said to be "nonlinear." A nonlinear system

can be almost linear, but there is no linear system which can be almost nonlinear. In general, linearity is a limiting case of nonlinearity. Therefore, any theory or technique adequate for a general nonlinear system is equally adequate for linear systems.

### Deterministic Linear Hydrologic Systems

The theory behind most linear methods can be generalized in the following manner. Suppose that  $s$  and  $\sigma$  are continuous variables representing position in space, and  $t$  and  $\tau$  define position in time. Consider the linear P. D. E. of the general form

$$L[g(s,t)] = f(s,t) \quad (17)$$

where  $L$  is linear P. D. operator of arbitrary order, and  $g(s,t)$  some function which satisfies equation 17 within a certain region  $R$ . Given the appropriate homogeneous boundary conditions along  $R$ , the solution of equation 17 can be written according to Hildebrand (1958) as

$$g(s,t) = \iint_R G(s,t; \sigma, \tau) f(\sigma, \tau) d\sigma d\tau \quad (18)$$

If

$$f(s,t) = f_s(s) f_t(t) \quad (19)$$

equation 18 can be written in the form

$$g(s,t) = \int f(\tau) \left[ \int_s G(s,t; \sigma, \tau) d\sigma \right] d\tau \quad (20)$$

If  $f_s(s)$  is spatially invariant, we may write

$$g(s,t) = \int_{-\infty}^{\infty} \tilde{G}(s,t; \tau) \tilde{f}(\tau) d\tau \quad (21)$$

where

$$\begin{aligned} \tilde{f}(\tau) &= f(s,t), \text{ and } \tilde{G}(s,t; \tau) \\ &= \int_s G(s,t; \sigma, \tau) d\sigma \end{aligned} \quad (22)$$

We can write equation 21 in differential equation form as

$$\begin{aligned} A_n(s,t) \frac{d^n g(s,t)}{dt^n} + A_{n-1}(s,t) \frac{d^{n-1} g(s,t)}{dt^{n-1}} + \\ \dots + A_0(s,t) g(s,t) = f(s,t) \end{aligned} \quad (23)$$



Usually, we are interested in the output variable at some particular point in space (a particular gaging station), in which case equation 23 becomes

$$A_n(t) \frac{d^n g(t)}{dt^n} + A_{n-1}(t) \frac{d^{n-1} g(t)}{dt^{n-1}} + \dots + A_0(t) g(t) = f(t) \quad (24)$$

which describes a spatially lumped parameter, time-varying linear system. If we assume that the parameters in equation 24 are time invariant, we obtain

$$A_n \frac{d^n g(t)}{dt^n} + A_{n-1} \frac{d^{n-1} g(t)}{dt^{n-1}} + \dots + A_0 g(t) = f(t) \quad (25)$$

which describes a time invariant, lumped parameter linear system. If we assume that the system is completely at rest at  $t=0$ , we can write equation 25 in the form of the convolution equation

$$g(t) = \int_0^t h(\tau) f(t-\tau) d\tau \quad (26)$$

which is the basis of the unit hydrograph theory and many other hydrologic techniques. In equation 26,  $g(t)$  represents runoff,  $f(t)$  rainfall, and  $h(t)$  is the kernel, or in this case, the unit hydrograph.

In summary, application of equation 26 implies that the watershed behaves as a linear system, it is time invariant, the rainfall is uniformly distributed over the watershed

area, and the watershed is completely at rest at the beginning of the rain. The "effective" precipitation is used as an input to the system. This implies that we know some method for the separation of the runoff hydrograph into base flow and direct runoff (fig. 9). This approach in fact is a combination, using system synthesis for the estimation of the effective precipitation, and system analysis for the estimation of runoff. Methods using this technique have been developed by Snyder (1955), Eagleson et al. (1966), Nash (1957, 1960), O'Donnell (1960), Dooge (1965), and others.

### Deterministic Nonlinear Hydrologic Systems

As it was noted in a previous section, a time invariant analytic system can be expanded in Volterra series. Such an expansion can be written in the form

$$y(t) = h_0 + \int_{-\infty}^{\infty} h_1(\tau_1) x(t-\tau_1) d\tau_1 + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1, \tau_2) x(t-\tau_1) x(t-\tau_2) d\tau_1 d\tau_2 + \dots + \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} h_n(\tau_1 \dots \tau_n) x(t-\tau_1) \dots x(t-\tau_n) d\tau_1 \dots d\tau_n + \dots \quad (27)$$

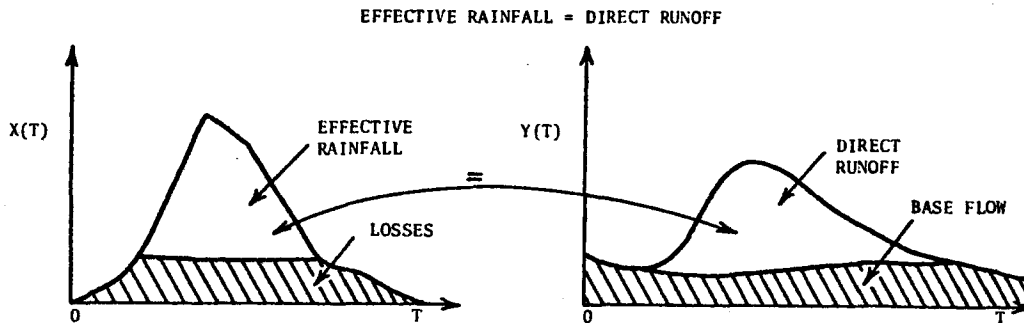


Figure 9. Estimation of effective rainfall through hydrograph separation.

where  $h_i$  are the kernels of the system and  $h_0=0$  unless a source or sink is present. If the system is physically realizable, and has finite memory (as it happens with hydrologic systems), equation 27 can be written in the form

$$\begin{aligned}
 y(t) = & \int_0^u h_1(\tau_1) x(t-\tau_1) d\tau_1 \\
 & + \int_0^u \int_0^u h_2(\tau_1, \tau_2) x(t-\tau_1) x(t-\tau_2) d\tau_1 d\tau_2 \\
 & + \dots \\
 & + \int_0^u \dots \int_0^u h_n(\tau_1 \dots \tau_n) x(t-\tau_1) \dots \\
 & \quad x(t-\tau_n) d\tau_1 \dots d\tau_n \\
 & + \dots
 \end{aligned} \tag{28}$$

subject to the condition

$$h_i(t) = 0 \text{ for all } \tau < 0$$

and where  $u$  is the length of the memory. If, instead of continuous functions, discrete sets of data are used, equation 28 can be written in the form

$$\begin{aligned}
 y(T) = & \sum_{S_1=0}^U H_1(S_1) X(T-S_1) \\
 & + \sum_{S_1=0}^U \sum_{S_2=0}^U H_2(S_1, S_2) X(T-S_1) \\
 & \quad X(T-S_2) \\
 & + \dots \\
 & + \sum_{S_1=0}^U \dots \sum_{S_n=0}^U H_n(S_1 \dots S_n) X(T-S_1) \\
 & \quad \dots X(T-S_n)
 \end{aligned} \tag{29}$$

The first term in equation 28 or 29 represents a linear system, that is an ordinary convolution integral. The other terms are a generalization of the convolution integral. In general, the  $i$ th term represents a pure subsystem of order  $i$ . Therefore our system  $y(t)$  is composed of the summation of a linear and a

series of nonlinear subsystems. If we represent the successive terms of the system by  $H_1(t)$ ,  $H_2(t)$ ,  $\dots$ ,  $H_n(t)$ ,  $\dots$ , respectively, the system can be written in the form

$$y(t) = H_1(t) + H_2(t) + \dots + H_n(t) + \dots \tag{30}$$

An illustration of a nonlinear system is given in figure 10.

A system is identified whenever its kernels  $h_n(t)$  are calculated. This evaluation is based on the past behavior of the system. Theoretically, after knowing all the kernels, the response of the system can be calculated for any given set of input values.

Equation 28 (in the form given) is very generalized and its usage is limited and time consuming. It is desirable to reduce the operations involved. For example, given a sequence of inputs, sometimes sequential values are highly correlated, and the system itself may smooth out rapid fluctuations. Hydrologic systems demonstrate both of these characteristics. Thus, we may look for ways which can describe the sequence by a smaller number of parameters. If the sequence  $x_1 \dots x_j \dots x_n$  represents observations at times  $t_1 \dots t_j \dots t_n$ , respectively, they can be approximated by a polynomial of the form

$$X_k(t) = \alpha_0 + \alpha_1 t + \dots + \alpha_k t^k = \sum_{m=0}^k \alpha_m t^m \tag{31}$$

where  $k < n$ , and such that

$$e_j = |X_k(t_j) - x_j|, (j = 1 \dots n) \tag{32}$$

is sufficiently small. Using such a polynomial approximation a number of difficulties arise. The most important is that the method becomes very sensitive to error when  $k$  exceeds 7 or 8 (Forsythe 1957). The use of orthogonal polynomials solves these difficulties. The normal equations of the least square data fitting completely decouple. The evaluation of the standard deviation employing the null hypothesis becomes much less time consuming. Fourier series expansion of a function falls into this category. Another approach is that of weighted polynomial expansion within an interval (the length of memory). If one takes as many polynomials as the length of

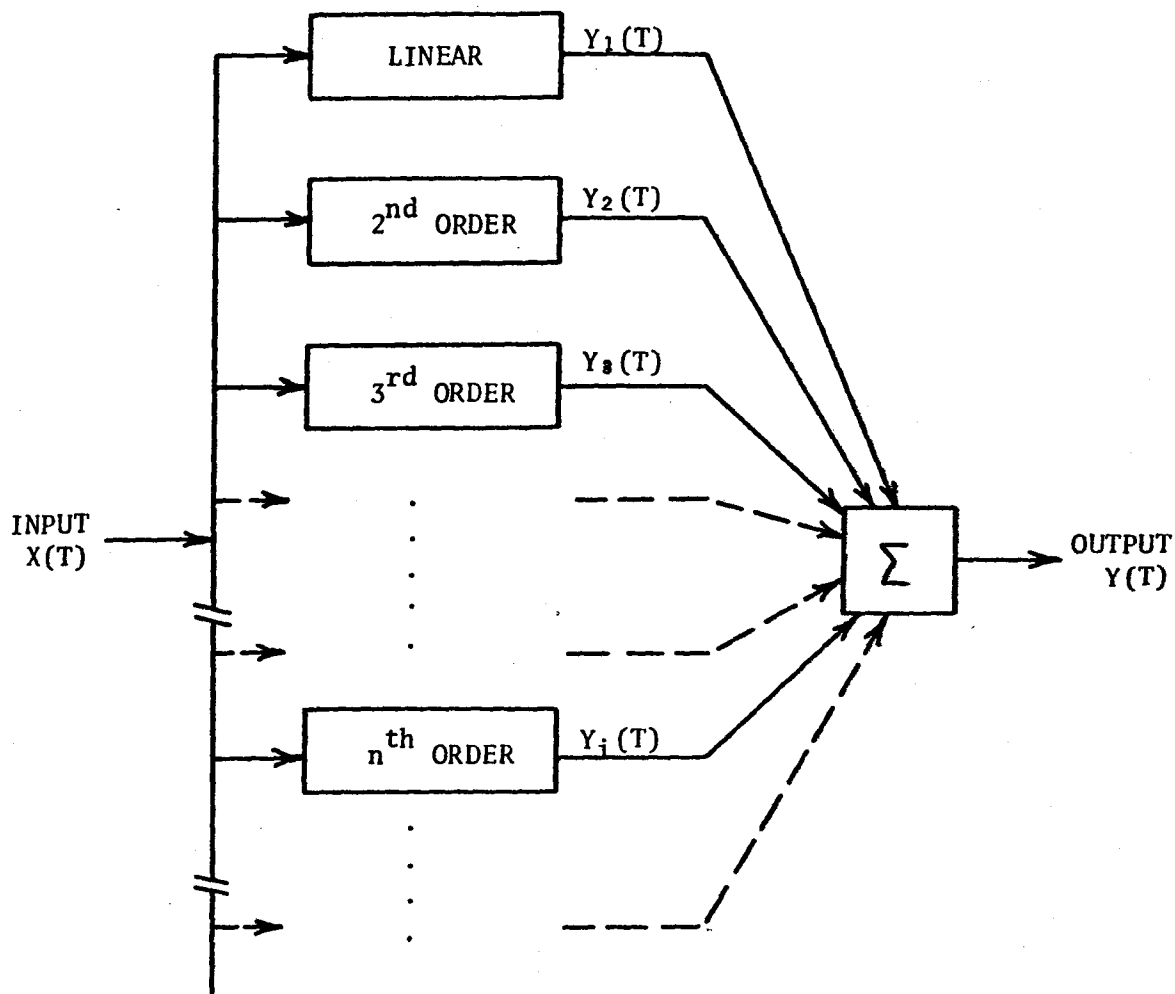


Figure 10. Illustration of nonlinear system representation by Volterra Series.

the memory, an exact match could be made to each of the input values. However, no economy in the description would have been affected. Instead, fewer polynomials can be used. This introduces an error, but due to the nature of the operation, it will be the least error possible.

Methods using the above procedure have been introduced by Jacobi (1966), Harder and Zand (1969), and Brandstetter and Amorocho (1970). It is this method which is being considered for use in this study. There are certain distinct advantages associated with the process. The method is quite general and can handle a variety of problems such as runoff, chemical quality of runoff waters, and suspended sediment predictions, when input values are properly weighted by functions describing the physical processes involved in

each case. Once the response functions or kernels of the system are evaluated, they may be used for predictions given any sequence of inputs. It requires a minimum amount of data, possibly 1 to 2 years of good records. Systems can be used in cascade, like

predict                  predict  
 precip. —————> runoff —————> quality or sediment

It can be used for quantitative evaluations of the changes created by any type of watershed management procedure by evaluating the kernels of the original and the managed hydrologic system. Emphasis will be given to establishing weighted functions for the best description of the physical process, and on deriving tools for the greatest economization of the procedure.

## Watershed Stratification: A Problem on Watershed 10<sup>4</sup>

We have also begun to structure the hydrologic model for watershed 10 concurrently with the simulation study underway for watershed 2 and the watershed systems analysis. Our objective here is to prepare a fine-resolution hydrologic model which incorporates the transpiration model from the Primary Producers, the evapotranspiration model from Meteorology, and provides soil moisture and subsurface water flow for the Primary Producer and Bio-Geochemical Processes groups. The first step in structuring such a model is system stratification.

One key to the analysis of complex systems is the compartmentalization of the system into homogeneous subsystems which can then

be isolated for study. This should be done in such a way that the linkage between compartments is simple and direct.

Another key is placement of compartment boundaries in such a way that the cells are easily uncoupled, or are coupled as a simple linear cascade. To do otherwise would complicate the modeling considerably. A hydrologic model that consists of a series of compartments arranged as a branching cascade is extremely difficult to manage. Water flow from an upper compartment must be somehow divided between lower compartments in the cascade. The basis for such division is generally obscure and usually arbitrary.

In our attempt to structure the hydrologic model for watershed 10 at the next level of resolution, we began by setting compartment boundaries along stream courses. This automatically decoupled the compartments, since water does not cross the channel (fig. 11).

Next, it was necessary to consider the arrangement of the plant communities within

<sup>4</sup> Authored by George W. Brown, Associate Professor, Oregon State University, Corvallis.

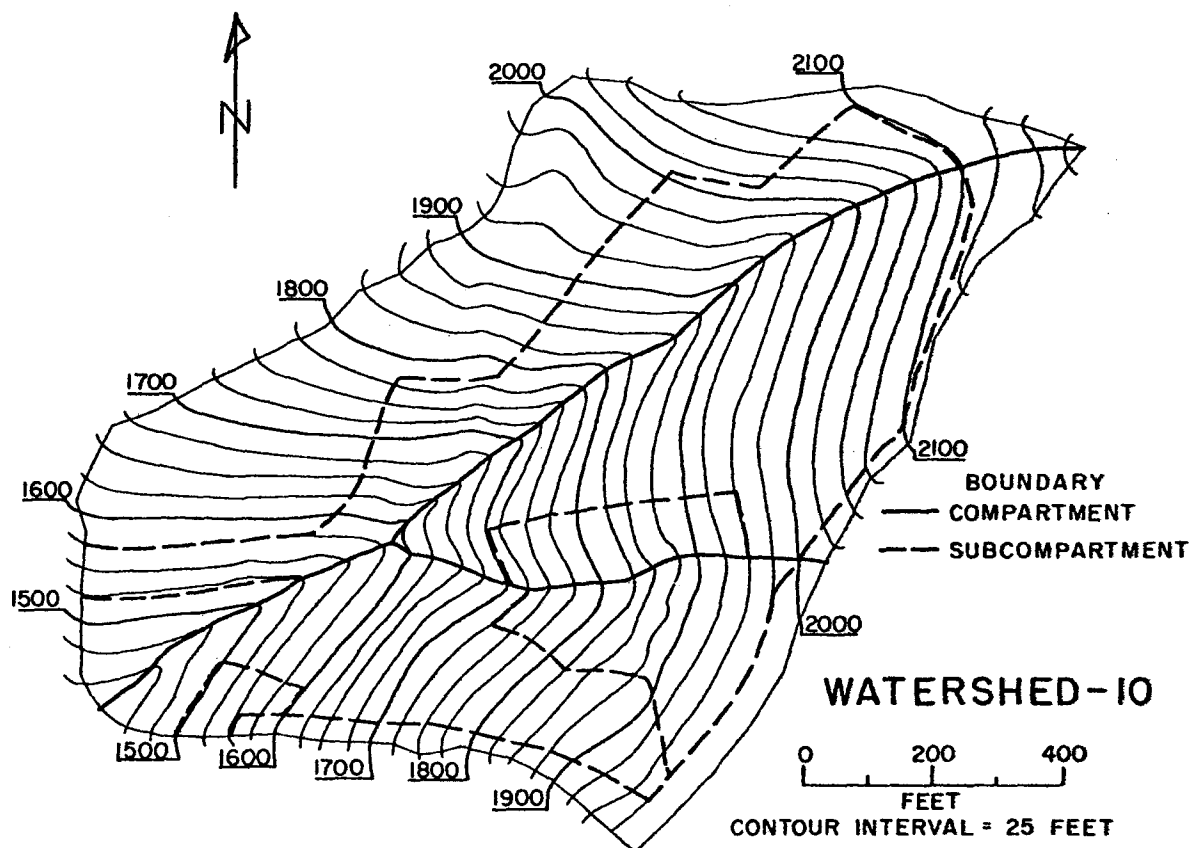


Figure 11. Initial and secondary stratification of watershed 10, H. J. Andrews Experimental Forest, Oregon.

the watershed. The hydrologic model and the primary producer model are obviously linked. The principal coupling variable between models is the soil moisture profile. Soil moisture is that portion of the "hydrologic state" of the watershed which closely regulates plant growth. Plants, in turn, influence soil moisture by transpiration. Thus, superimposition of the primary producer's vegetative structure upon the structure of the hydrologic system is essential. This structure was combined with the initial hydrologic stratification and defined the subcompartment boundaries. Subcompartment boundaries approximate the vegetative type-map boundaries and are arranged into riparian, midslope and ridgetop zones. These zones undoubtedly reflect the changes in soil moisture regime within the watershed. Also, this stratification allows us to consider flow between subcompartments as simple linear cascades.

This final stratification for watershed 10 will provide the basis for sampling schemes to characterize soil moisture, water flow and other hydrologic and biologic processes necessary for the next round of model construction. It is essential to note that this stratification is compatible for modeling hydrologic processes and is also compatible for linking the hydrologic model with that of the primary producers. It is the major achievement of our initial modeling effort and sets the stage for continued progress.

### *Acknowledgments*

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